TWO ESSAYS ON PRODUCT AND PRICING INNOVATION

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Abstract

My dissertation adopts analytical method to understand how firms innovate their marketing activities in response to the constantly changing business environment, and how such activities impact the behavior and wellbeing of consumers. In essay 1, I develop a profit-enhancing selling method, *contingency selling*, that is tailored to the unique features of sports events marketing, including capacity constraints, game uncertainty, and preference heterogeneity. I find that when the capacity constraint is tight, contingency selling outperforms other selling mechanisms such as traditional selling with or without a secondary market, spot selling, and ticket options. Essay 2 focuses on "all-or-nothing", a unique feature of crowdfunding. I show that when the new product's market potential is uncertain, the "all-or-nothing" feature provides double benefits to the entrepreneur. First, it safeguards the entrepreneur against the downside market uncertainty. Second, this safeguard enables the entrepreneur to take advantage of the upside uncertainty by producing products of higher quality, which are typically associated with higher risk. Jointly, these two effects lead to the counter-intuitive result that that crowdfunding allows the entrepreneur to benefit from high degree of uncertainty.

Introduction

My research focuses on innovations in product development and pricing. I apply theories and methods from marketing and economics to phenomena and practices, where innovations are crucial, including sports markets and crowdfunding. In these contexts, I identify the key institutional features that make the context unique, and examine the impact of these features on market participants such as managers and consumers.

For example, in my dissertation study on crowdfunding, I focus on "all-or-nothing", which I describe as the key feature that distinguishes crowdfunding from the conventional way of funding and selling products. I show that this feature has considerable impact on both entrepreneurs' strategy (essay 1) and consumers' collective actions (essay 2). Similarly, in a separate paper being revised for Management Science, I develop a profit-enhancing selling method that is tailored to the unique features of sports events marketing, including capacity constraints, game uncertainty, and preference heterogeneity. The following sections elaborate on how I implemented my research philosophy in various research projects.

In essay 1, I propose a new pricing model for markets where the product is sold in advance with attribute uncertainty and capacity constraint. A typical example is tournament ticket sales for sports events where it is uncertain whether a particular team will appear in the game. The model enables the firm to sell tickets in a contingent fashion—i.e., some tickets will be valid only if a certain team gets into the game. When consumers (fans) have sufficiently different preferences for different products (games with different teams), selling these "contingent tickets" better matches the price with the willingness-to-pay of particular consumer segments (fans of specific teams). By allowing for both state-based and belief-based price discrimination, contingent tickets generate a higher margin per seat for the firm than the traditional way of selling general tickets. This is especially true when the firm faces acute supply constraints. I further show that the proposed pricing model for advance selling continues to be advantageous in the presence of secondary markets where consumers purchase tickets in advance but can trade tickets once they know which team will be in the game. I discuss managerial implications of the contingency pricing model for advance selling and how it can be implemented in practice.

The context of my essay 2 is reward-based crowdfunding platforms (e.g., Kickstarter), which have become a popular choice by entrepreneurs. I examine how quality, pricing and advertising strategies in crowdfunded platforms differ from those in the traditional model of market entry where firms undertake market research; develop, produce and sell the product through a distribution channel. The key aspect of crowdfunding is the All-Or-Nothing (AON) feature, in which an entrepreneur does not produce or sell anything unless the project goal is met. This feature provides demand safeguards and leads to two important benefits for the entrepreneur. First, crowdfunding safeguards against poor market conditions. I call this effect the direct effect of crowdfunding. Second, demand safeguard allows the entrepreneur to advertise more, produce a higher quality, charge a higher price and make higher profit. I call this the indirect effect of crowdfunding. In equilibrium advertising and quality decision are strategic complements. In further exploring the implications of market size uncertainty, interestingly, my results suggest that safeguarding benefits enable the entrepreneur to receive higher profit when the uncertainty is higher.

In the remainder of this dissertation, I present the two essays in sequence. The tables are presented in a separate chapter due to their large sizes. The Appendices at the end of the dissertation contain all the technical proofs for these two essays.

Essay 1: Service Pricing with Capacity Constraint: A Model for Sports Markets

1. Introduction

As multi-billion-dollar businesses, sports markets present some unique challenges to research and practice. Most popular sports, whether professional or college football in the U.S. or soccer championship games in Europe and South America, include tournament style events in which only the winning teams advance to the next rounds. For instance, every year only two teams play in the Rose Bowl of American college football or the Super Bowl of the National Football League (NFL). Two characteristics related to ticket selling for such events are worth noting. First, the event organizer pre-determines the venue and sells tickets well in advance of the actual event happening (Geng et al. 2007, Sainam et al. 2010). A buyer, therefore, engages in advance purchase of tickets without knowing whether his or her favorite team will make it to the game. Thus, the valuation of the ticket is ex-ante uncertain and depends on the buyer's assessment of the odds that his or her favorite team will appear in the game. A second issue relates to the constraint in the number of seats available. Simply put, the demand for seats often exceeds the service capacity of the venue. For example, when it comes to the Super Bowl, "we can never meet the demand for tickets," said NFL spokesman Brian McCarthy, "If I could build a stadium for 1 million people I still wouldn't have enough available tickets." (USA Today, 2013).[1]

In this paper, I examine the problem of ticket pricing where the event organizer (thereafter referred to as "the firm") engages in advance selling while facing a service capacity constraint. I propose a contingency selling model and assess its properties in contrast to the traditional approach. In the case of traditional selling, the firm sells "general" tickets that allow admission to the game, no matter which teams play. In the proposed contingency selling, the firm sells "contingent" tickets that are only good for redemption if a particular team makes it to the game. my analysis suggests that compared to traditional selling, contingency selling benefits the firm to a greater extent when consumer preferences are more heterogeneous, or the service capacity is more limited.

[1] http://www.usatoday.com/story/news/nation/2013/05/19/super-bowl-ticket-lottery/2322955/, accessed on 7/1/2017.

In the sports industry, several start-ups have experimented with contingency selling. A pioneer was Yoonew.com, an online ticket service founded in 2004 by two MIT students. Covering games for the NCAA's college basketball's Final Four, NFL, and MLB (Major League Baseball), Yoonew sold ticket futures for a particular team (or a match between two particular teams), much like futures for stocks and goods in financial markets. If the team gets to the game, the holder of the ticket future can redeem it for the actual ticket. If the team does not get to the game, the ticket future becomes unredeemable and its value is lost. The novelty of the Yoonew business generated much enthusiasm and media attention. As the New York Daily News excitedly claimed, for anyone who could not obtain a ticket to the Super Bowl, "There's always next year and yoonew.com."[2]

Yet, Yoonew faced two main challenges in the implementation of its selling model. First, like many start-ups, the company lacked trust and familiarity with consumers, which is critical when they need to pay hundreds or thousands of dollars in advance for a ticket. Second, being a broker itself, Yoonew had to purchase actual tickets from the market for redemption and, not surprisingly, often found it difficult to do so. It thus had to establish relationships with larger ticket brokers and season-ticket holders to mitigate this risk. Despite these challenges, Yoonew successfully operated until 2010 when the bank responsible for its merchant account went bankrupt amid the financial crisis, causing severe cash flow problems for the company.

Nevertheless, the core idea of selling tickets contingent on future states has strong appeal and continues to be experimented and discussed in the industry. For example, starting August 8, 2017, College Football Playoff (CFP) has allowed fans nationwide to make team-based ticket reservation to the 2018 CFP National Championship through the website (https://cfp-rsvp.com/).

In this paper, I formulate and formally study the properties of this new pricing method, taking into account three key characteristics of the market—product uncertainty, capacity constraint, and heterogeneous consumer preferences. It is important to point out that the contingent tickets in my

[2] http://www.nydailynews.com/sports/football/giants/missing-72-dolphins-patriot-perfection-somethin-article-1.343533, accessed on 10/11/2017

pricing model are sold by the event organizer rather than by brokerage services like Yoonew. This directly eliminates the two challenges Yoonew faced: trust and the availability of tickets.

We outline the idea behind contingency selling with an illustrative example. Let me consider the simple case of two teams, A and B. Both teams have a chance of advancing to a future tournament game. The firm sells tickets for the game in advance to a total of two consumers (or segments): a fan of team A and a fan of team B. The fans' valuation of the tickets depends on the value of watching their favorite team play and their assessment of whether the favorite team will make it to the game. For the ease of exposition, assume the chances that team A and B enter the game (call these states A and B) are equal, i.e., $Prob(A) = Prob(B) = 0.5$, and the valuation of watching the favorite team versus a less favorite team play is \$10 versus \$2. Both fans hold the correct belief about the two teams' chance of entering the game. These assumptions are summarized in Table 1 on page 62.

In traditional selling with risk neutral consumers, the firm charges \$6 for a general ticket. If the stadium capacity for these consumers can accommodate both of them, then the firm sells to two consumers and makes \$12. However, what happens if there is a supply side constraint? For instance, if the stadium can accommodate just one consumer, the revenue drops to \$6 regardless of which consumer ends up with the ticket. Limited capacity reduces sales even though the potential demand remains at two.

Now consider contingency selling. In this new approach, the firm will sell two contingent tickets— an "A ticket" and a "B ticket." The A ticket will be valid if team A advances to the game but will be invalid if team B advances. Vice versa for the B ticket. Since each consumer believes her favorite team has 50-50 odds to enter the game, she can pay \$5 for a contingent ticket associated with her favorite team. Total sales are two and the firm makes \$10. Notice, however, the firm is still able to sell two contingent tickets irrespective of whether there is a capacity constraint at one. This generates two observations. When there is no capacity constraint, traditional selling generates higher revenue than contingency selling (\$12 versus \$10). However, when there is a capacity constraint, contingency selling increases (\$10 versus \$6).

The intuition of this example lies in the fact that the consumers have different preferences for the two states (i.e., which team ends up playing). With traditional selling, the two states are "sold" together and thus indivisible from each other. *A* general ticket forces consumers to pay for not only their preferred state but also for their less preferred state. Traditional selling essentially bundles the states and, depending on capacity constraint, the firm makes either \$12 or \$6. In contrast, contingency selling decouples the states and allows the firm to charge each state the price (\$5) that fully extracts consumers' willingness to pay for their preferred state while, at the same time, diminishes the impact of capacity constraint. In a nutshell, contingency selling maximizes margin per seat through "interstate price discrimination" when there is capacity constraint.

In my formal model, I generalize the illustrative example and examine situations where contingency selling can generate higher revenue than traditional selling, and allow consumers to hold heterogeneous beliefs about the teams' winning odds. In an important extension, I also account for the possibility that consumers can trade among themselves in the secondary market. I analyze two distinct pricing mechanisms in the secondary market—a neutral pricing scheme and a decentralized scheme using Rubinstein infinite-period bilateral bargaining.

The remainder of the essay is organized as follows. In Section 2, I review the related literature in marketing, economics, and operations research. Section 3 develops the selling model for contingency selling. Section 4 incorporates a secondary market into the model. Section 5 compares contingency selling to alternative selling mechanisms such as spot selling. I conclude the paper with summaries and implications.

2. Literature

The proposed pricing model for advance selling is related to two streams of literature. First, my work contributes to the research on advance selling as a general pricing practice (e.g. Gale and Holmes 1992; Gale and Holmes 1993; Biyalogorsky and Gerstner 2004; Biyalogorsky et al. 1999; Fay and Xie 2010; Shugan and Xie 2000, 2005; Xie and Shugan 2001; Prasad et al. 2011; Cho and Tang 2013; Yu et al. 2015). The majority of these studies focus on the comparison between advance selling and spot selling; that is, selling tickets after the uncertainty about the desirability of the product is resolved. For example, Shugan and Xie (2000; 2005) find that advance selling is profitable when the desirability of the product is unknown to both the seller and the buyer in advance, but known to the buyer at the time of consumption. Xie and Shugan (2001) show that advance selling can be a better strategy than spot selling when the capacity is large. Prasad et al. (2011) further show that advance selling is preferred only when the marginal cost of production is low. Yu et al. (2015) find that when consumers' valuations of the product are highly correlated with each other, advance selling is a more profitable strategy than spot selling. Finally, Fay and Xie (2010) show that by offering consumers a choice with uncertainty (i.e., creating a probability good), advance selling can increase sales.

Our study differs from this literature in several ways. While I consider advance selling as the context for both traditional selling and contingency selling, the focus of my paper is to propose contingency selling as a specific implementation of advance selling and show it can generate greater revenue for the firm. Furthermore, capacity constraint is not a central issue in much of the advance selling literature. However, it is a primary market condition for which I develop my contingency selling model. Finally, I model the impact of secondary market, which has not been considered in this literature.

The other stream of research related to my paper includes the studies on ticket pricing and secondary markets. Cui et al. (2014) provide an excellent review of this literature. As they show, a key issue that extant studies address is whether allowing the resale of tickets (i.e., the existence of secondary markets) benefits or hurts the firm (e.g., Courty 2003; Möller and Watanabe 2010; Geng et al. 2007; Karp and Perloff 2005; Su 2010; Lim and Tang 2013). For example, Courty (2003) shows that, by allowing resale, a monopolistic firm can do equally well in advance selling as in spot selling. Geng et al. (2007) find that if there are only a small number of high valuation consumers, firms can obtain higher sales by allowing consumers to resell tickets before the event. Some other papers have considered the coexistence of regular consumers whose sole interest is to attend the event and professional scalpers who try to make money from the resale market (Karp and Perloff 2005; Möller and Watanabe 2010; Lim and Tang 2013). In contrast to these studies, my purpose is to show that, regardless of the existence of secondary markets, contingency selling can always benefit the firm relative to traditional selling when service capacity is limited. Furthermore, most extant studies do not model consumer heterogeneity regarding which “state” is preferred. I take this important characteristic into consideration and show that the heterogeneous preference for teams plays a key role in the implementation of contingency selling. In addition, even though the secondary market is considered, previous research has not paid attention to how prices are negotiated between buyers and sellers. I analyze both a neutral pricing scheme and a decentralized pricing scheme, and show the optimality of traditional selling versus contingency selling for each scheme.

Finally, some ticket pricing research has examined ticket options, a selling method that allows consumers to pay a fee for the option to buy a ticket at a later time. If the consumer wants to redeem the option, she needs to pay an execution fee for the ticket. In the paper by Cui et al. (2014), the authors first examine whether preventing the resale of tickets benefits the firm. They then link ticket options to resale and show that options can generate greater revenue for the firm by reducing ticket resale. They also show that with ticket options, the firm can benefit further if resale is prohibited altogether. These findings and theoretical insights are very different from ours. I demonstrate that contingency selling is preferred to traditional selling regardless of resale. The mechanism is that contingency selling better matches each seat in the capacity-constrained venue with the consumer who values it to a greater extent, taking into account the uncertainty about which team will be playing.

Sainam et al. (2010) also study ticket options. They show that options are profitable when the firm faces both team-based consumers (i.e., consumers who gain value only if their favorite team plays in the game) and game-based consumers (i.e., consumers who gain value from watching any games). my paper differs from their study in three important ways. First, Sainam et al. (2010) do not consider the capacity constraint, which is an important feature of sports events that motivates my research and is a key feature of my model. Second, they examine the pricing of tickets that target the fans of one team. The model is thus applicable in managing revenue at the level of an individual team. I model consumer heterogeneity differently and allow different consumers to favor different teams. Such heterogeneity is typical of sports markets and applies to revenue management at the league level. Third, Sainam et al. (2010) do not consider secondary market, but I allow for the possibility of ticket resales and examine how it influences my results.

3. The Main Model

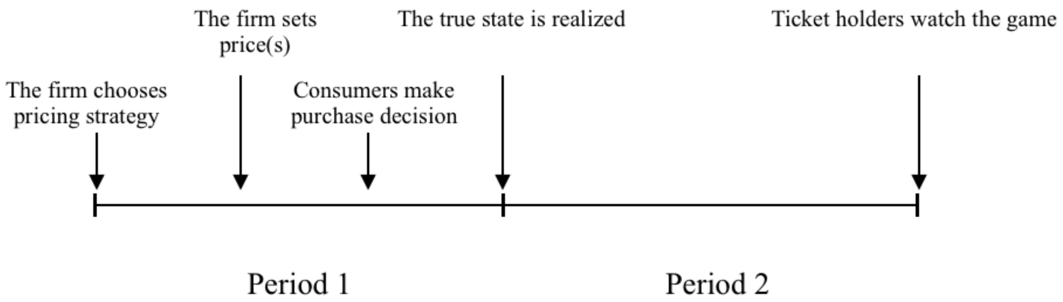
We consider a firm and two consumer segments (1, 2), each of size 1, participating in a two-period game. Similar to the numerical example described earlier, there are two possible states with A (B) representing the scenario that team A (B) ends up playing in the game for which the tickets are being sold.[3] The probability of state A occurring is ρ, which is unknown in advance. I allow consumers to hold heterogonous beliefs about ρ. This not only captures the uncertainty in the market but also accounts for the fact that some consumers are more optimistic about their team while others are less so. I assume the beliefs follow the uniform distribution on [0,1].

Consumers in segment 1 prefer state A over state B. Segment 2 consumers have the opposite preference. Consumers have a reservation value V for their preferred state and βV for the less preferred state, $\beta \in (0, 1)$. Parameter β is a (reversed) measure of preference heterogeneity for the states: higher β implies that valuations for states A versus B are more similar, thus less heterogeneity in preference. Without loss of generality, V is normalized to be 1.

In the first period, which state will occur is uncertain. The firm first chooses whether to sell general tickets under traditional selling or to sell contingent tickets under contingency selling. It then sets the price(s) correspondingly. Based on ticket price(s), consumers make their purchase decisions. In traditional selling, the firm sets a price P for the ticket and sells a quantity of $min\{Q, SC\}$, where Q is the potential demand and SC is service capacity. In contingency selling, the firm charges price P_S for a contingent ticket and sells $min\{Q_S, SC\}$ for each type of ticket, $s = A, B$. In the second period, one of the two states occurs and ticket holders can use their tickets accordingly. Figure 1 presents the decisions and the game structure.

Figure 1. Decisions and Game Sequence in the Main Model

[3] For parsimony, our analysis focuses on one side of the tournament with two teams competing for one slot in the game. There are of course teams competing for the other slot. Our model considers one side but the two sides are symmetric. It can be shown that, given capacity constraint, contingent ticket prices do not change even when one considers both sides of the tournament.



In my main model, the firm engages in advance selling by setting prices and selling tickets before the uncertainty associated with the product (game) is resolved. I focus on the comparison of traditional selling versus contingency selling in advance selling for several reasons. Advance selling is the prevalent selling practice for major sports events. Tickets are often available for purchase at the beginning of the season, long before it is known which teams will be playing in certain games. For the event organizer and the sports league, advance selling generates an early revenue stream. It also provides a valuable revenue cushion by reducing the risk associated with less popular teams ending up playing the game. Moreover, advance selling is a key mechanism to engage fans early. Enthusiasm from the fanbase makes it not only possible but often imperative to sell tickets early. In fact, pricing and selling tickets after the uncertainty is resolved is viewed by many fans as unfair practice (Sainam et al. 2010). This is an important reason why the major sports leagues engage in advance selling (Happel and Jennings 1995; Krueger 2001). Finally, advance selling is an effective promotional tool that generates awareness and interest for the event among fans and the general public.

Therefore, my main model focuses on how contingency pricing works for advance selling when there is product uncertainty coupled with capacity constraint, both with and without secondary market. I provide several robustness checks, including the case of spot selling, in Section 5 to demonstrate that the advantage of my pricing model persists.

In the analysis below, I first outline the solutions when the firm faces no capacity constraint and is able to offer more seats than the total demand from both segments. This is essentially the case of $SC > 2$. After this benchmark case, I analyze the situation with the capacity constraint of $0 < SC \leq 2$. The model is extended in Section 4 to count for secondary market.

3.1. No Capacity Constraint (SC > 2)

Under traditional selling, the firm sells general tickets that are valid regardless of the true state. For a consumer in segment 1, her valuation for a general ticket is $\rho + (1-\rho)\beta$. Since heterogeneous beliefs ρ fall within [0,1] uniformly, the valuations by segment 1 consumers follow the uniform distribution on [β,1]. Similarly, segment 2 consumers' valuations also follow the uniform distribution on [β,1]. These two consumer segments jointly generate the following aggregate demand:

$$Q_T(P) = \begin{cases} 2, & P < \beta \\ \dfrac{2(1-P)}{1-\beta}, & \beta \le P < 1 \\ 0, & P \ge 1 \end{cases} \tag{1}$$

Naturally, the profit function is $\pi_T(P) = Q_T(P)$. It is straightforward to show that the firm's optimal price is $P^* = \frac{1}{2}$ if $\beta < \frac{1}{2}$, and $P^* = \beta$ otherwise. The former generates a profit of $\frac{1}{2(1-\beta)}$ and the latter generates a profit of 2β. Please refer to Table 2a on page 62 that summarizes the equilibrium price, quantity sold, and revenue.

Now consider the case when the firm sells contingent tickets. Consumers' willingness to pay for a contingent ticket depends on their valuations for the associated state and their beliefs about the probability of that state occurring. For instance, a segment 1 consumer's valuation for ticket A is ρ, while a segment 2 consumer's valuation for ticket A is $\rho\beta$. Since ρ is distributed uniformly in [0,1], segment 1 consumers' valuations are uniformly distributed in [0,1], while segment 2 consumers' valuations are uniformly distributed in [0, β]. Total demand is as follow:

$$Q_A(P_A) = \begin{cases} 2 - \dfrac{1+\beta}{\beta} P_A, & P_A < \beta \\ 1 - P_A, & \beta \le P_A < 1 \\ 0, & P_A \ge 1 \end{cases} \tag{2}$$

The corresponding profit function is $\pi_A(P_A) = Q_A(P_A)P_A$. The firm's optimal price is $P_A^* = \frac{1}{2}$ if $\beta < \frac{1}{3}$, and $P_A^* = \frac{\beta}{1+\beta}$ otherwise. In a similar fashion, one can derive the optimal price for the contingent tickets for state B. Table 2b reports the price, quantity sold, and revenue. The comparison of firm revenue between traditional selling and contingency selling generates the following proposition:

Proposition 1.1. When the capacity is unlimited, traditional selling dominates contingency selling.

The intuition behind Proposition 1.1 builds on the aforementioned differences between traditional and contingency selling. On the one hand, under traditional selling, the firm charges a price to capture consumers' expected valuation for the event. Such a price can potentially expand demand because it couples the two states together so that the ticket is attractive to an average consumer. On the other hand, contingency selling is more effective at maximizing the margin per seat because it sells each contingent ticket to its high valuation segment. When the capacity is not an issue, traditional selling outperforms contingency selling due to its advantage in demand expansion.

3.2. Constrained Capacity (0 < SC ≤ 2)

Consumer valuation of tickets, regardless of general or contingent tickets, does not change with respect to the capacity constraint. However, capacity constraint (SC) influences how many consumers can be admitted to the venue. Consider the case of traditional selling, the demand now takes the following form:

$$Q(P) = \begin{cases} SC, & P < 1 - \dfrac{SC(1-\beta)}{2} \\ \dfrac{2(1-P)}{1-\beta}, & 1 - \dfrac{SC(1-\beta)}{2} \leq P < 1 \\ 0, & P \geq 1 \end{cases} \tag{3}$$

When SC is smaller than $\frac{1}{1-\beta}$, the firm's best strategy is to charge $P = 1 - \frac{SC(1-\beta)}{2}$, selling up to its full capacity SC for a revenue of $SC\left(1 - \frac{SC(1-\beta)}{2}\right)$. When $SC \geq \frac{1}{1-\beta}$, the firm sells to $\frac{1}{1-\beta}$ consumers by pricing at 1/2. The revenue is $\frac{1}{2(1-\beta)}$. A similar analysis can be done for contingency selling. Please refer to Table 3 on page 63 and Table 4 on page 64 for the results. The Appendices contain the derivations of the results in Table 3 and 4 and all other technical results. The comparison generates the following finding:

Proposition 1.2a. With capacity constraint, contingency (traditional) selling generates higher revenue if $SC < \overline{SC}$ ($SC \geq \overline{SC}$). $\overline{SC}$ equals $\frac{1-\sqrt{\beta}}{1-\beta}$ for $\beta < \bar{\beta} \approx 0.420$ and $\frac{2(3\beta-1)}{-1+4\beta+\beta^2}$ otherwise.

In contrast to Section 3.1, capacity constraint is a key issue and contingency selling becomes optimal when SC is low. To see this result, consider the low capacity case ($SC < \frac{1}{2}$). As shown in Table 3, with traditional selling, the maximum price the firm is able to charge is $1 - \frac{SC(1-\beta)}{2}$. At this price, $\frac{SC}{2}$ of the segment 1 consumers and $\frac{SC}{2}$ of the segment 2 consumers get served. However, with contingency selling (Table 4), the firm sells the A ticket at a price of $\max\left\{1 - SC, \frac{\beta(2-SC)}{1+\beta}\right\}$ to SC consumers. Also, it sells the same number of B tickets at the same price to SC consumers. Altogether, the firm is able to charge a price of $\max\left\{2 - 2SC, \frac{2\beta(2-SC)}{1+\beta}\right\}$ for each available seat. Therefore, by issuing contingent tickets, the firm sells the same number of seats to both consumer segments and, in turn, earns a higher margin per seat than traditional selling, i.e., $\max\left\{2 - 2SC, \frac{2\beta(2-SC)}{1+\beta}\right\} > 1 > 1 - \frac{SC(1-\beta)}{2}$.

When supply constraint is less of an issue, however, traditional selling turns out to be the better strategy. Consider the case when SC is close to 2. The optimal price and sales for traditional and contingency selling are close to the no capacity constraint case.

These results indicate that contingency selling benefits the firm by allowing it to extract greater margin per seat when it faces the capacity constraint. This is achieved by decoupling the states so that each segment of consumers buys the contingent ticket corresponding to its preferred state. More critically, however, by decoupling the states, the firm is able to sell the available seats to both segments of consumers. This strategy is more effective when the capacity becomes more limited.

As shown in Appendices, the threshold $\overline{SC}$ is a function of β, i.e., the heterogeneity in consumer preference. Proposition 1.2b summarizes how $\overline{SC}$ is affected by β:

Proposition 1.2b. When consumer preference heterogeneity is high ($\beta \leq \bar{\beta}$), $\overline{SC}$ is decreasing in β. When consumer preference heterogeneity is low ($\beta > \bar{\beta}$), $\overline{SC}$ is increasing in β.

The proof is included in the Appendices. Proposition 1.2b shows the nuanced impact of consumer preference heterogeneity: the region of SC over which contingency selling outperforms traditional selling is first decreasing and then increasing in β. To understand this non-monotonicity, it is useful to examine the different impacts of β on the two selling mechanisms.

When consumer preference heterogeneity is high (i.e., β is small), under contingency selling, the firm sells contingent tickets of each state only to the consumer segment who values the state. In the example of an A ticket, the firm sells it only to the fans of team A. With small β, fans of the other team are not lucrative enough for the firm to target. Therefore, a marginal increase in β does not influence the firm's revenue. However, under traditional selling, since the two states are bundled together, a marginal increase in β enhances every consumer's valuation for the general ticket and, in turn, increases the revenue. Thus, at low values of β, a marginal increase in β boosts the revenue for traditional selling but not for contingency selling. As a result, the range of SC over which contingency selling dominates traditional selling shrinks.

However, when β is large, the difference in valuations between a consumer's preferred and less preferred states becomes smaller. Under both selling mechanisms, the firm ends up selling tickets of each state to serve consumers of both segments. When this happens, the effect of β is moderated by the belief parameter ρ. Consider the case of A tickets under contingency selling. The firm sells A tickets not only to segment 1 consumers (A fans) but also to segment 2 consumers (B fans) whose ρ is high. Recall that segment 1 consumers' valuation for ticket A is ρ, while segment 2 consumers' valuation is $\rho\beta$. Thus, a marginal increase in β enhances segment 2 consumers' valuation by ρ but it does not influence segment 1 consumers, and the effect of β occurs only among segment 2 consumers whose ρ is high. Similarly, for B tickets, a marginal increase in β boosts the valuation of segment 1 consumers by $1-\rho$ but it does not influence segment 2 consumers. The effect of β occurs among segment 1 consumers who have high $1-\rho$, or equivalently low ρ.

In the case of traditional selling, the valuation for the general ticket is $\rho+(1-\rho)\beta$ for segment 1 consumers and $\rho\beta+1-\rho$ for segment 2 consumers. A marginal increase in β increases the valuation of segment 1 consumers by $1-\rho$ and segment 2 consumers by ρ. Due to the capacity constraint, the firm only serves segment 1 consumers whose ρ is high and segment 2 consumers whose

ρ is low since they value the general ticket to a greater extent than others. In other words, the effect of β works through the segment 1 consumers whose ρ is high and the segment 2 consumers whose ρ is low.

It is therefore clear that, when β is high, under both selling mechanisms, the marginal effect of β is $1-\rho$ for segment 1 and ρ for segment 2. However, what kinds of consumers are the base for the marginal effects differ between the two mechanisms.

Let me look at segment 1 consumers first. Under contingency selling, an increase in β helps enhance firm revenue through affecting segment 1 consumers' valuation of B tickets and, as shown above, this enhancement only occurs on those with low ρ (thus $1-\rho$ is high). Under traditional selling, however, an increase in β helps increase revenue through segment 1 consumers whose ρ is high (thus $1-\rho$ is low). Thus, based on segment 1 consumers, the magnitude of the marginal effect of β, i.e., $1-\rho$, is higher under contingency selling than under traditional selling. Similarly, based on segment 2 consumers, the marginal effect of β is also stronger under contingency selling than under traditional selling.

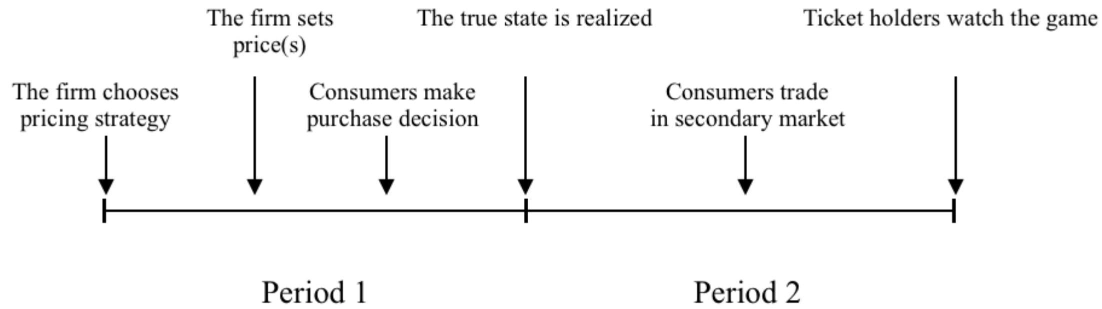
Taken together, considering both segments, a marginal increase in β amplifies the advantages of contingency selling over traditional selling. This explains an increasing $\overline{SC}$ when β is large.

Propositions 1.1 and 1.2 suggests that the efficacy of interstate price discrimination through contingency selling is moderated by two market primitives—consumer preference heterogeneity β and service capacity SC. While β has a non-monotonic impact on the comparison between contingency and traditional selling, lower SC always improves the efficacy of contingency selling as fully extracting the rent from each seat becomes more valuable.

4. Secondary Market

We now consider a more general framework wherein customers can trade tickets with each other in the second period (Figure 2). At this point, which team is playing in the game becomes known to the public. Consumers who hold tickets can resell if their favorite team does not make it to the game and those who do not have a ticket can buy if their favorite team makes it to the game. Trading in secondary markets among consumers is common in some sports markets. According to Technavio (2015), the value of the secondary ticket market in 2015 was \$8.94 billion, 54.73% of which came from sports tickets. The revenue of StubHub, the world's largest ticket resale website, has grown from \$100 million in 2006 to \$725 million in 2015.

Figure 2. Decisions and Game Sequence with Secondary Market



In the secondary market, some consumers become buyers if they do not currently own a ticket and they are willing to pay as much as 1 to acquire the ticket. At the same time, some consumers become sellers if they hold tickets but their preferred state did not occur. They are willing to sell their tickets for anything above β. I use upper case P to represent the price set by the firm in the first period and lower case p to denote the resale price in the secondary market which, as I outline below, is negotiated between potential buyers and sellers. Whether a consumer becomes a buyer or seller in the secondary period depends on whether she gets a ticket in the first period. I assume the following rationing rule: when the demand exceeds the capacity $(Q > SC)$ in the first period, all potential buyers who find the price acceptable have an equal chance to get the ticket. When x consumers from segment 1 and y consumers from segment 2 are willing to buy a ticket, then $\frac{x}{x+y}SC$ tickets go to segment 1 and $\frac{y}{x+y}SC$ tickets go to segment 2. The firm does not participate in the secondary market.[4]

When making purchase decisions in the first period, consumers are aware that there will be a secondary market. The impact of a secondary market on consumers' decision making is thus twofold. On the one hand, the existence of a secondary market can push consumers to "buy now" since they have

[4] Anecdotally, the assumption that the firm does not participate in the secondary market is supported by the fact that sports leagues, such as the NFL, do not buy back and resell tickets that they have already sold, although they do not preclude consumers from directly participating in trades through secondary markets such as StubHub.com. From a technical standpoint, this assumption helps keep the model parsimonious so that the impact of the secondary market on contingency selling can be parsed out more clearly.

the opportunity to resell later if their preferred state is not realized. On the other hand, a secondary market also reduces the motivation of advance purchase since it is possible to wait to buy a ticket from the secondary market. In other words, the presence of a secondary market may motivate consumers to "buy later." Whether a customer buys now or buys later depends on both the ticket price set by the firm and the price she expects in the secondary market. The firm needs to consider these consumer decisions when it sets the ticket price in the first period. Thus, the first step is to model how the resale price in the secondary market is determined.

In principle, any price between buyers' and sellers' valuations of the ticket can clear the secondary market. The actual price, however, depends on how they agree to split up the surplus generated from the trade. I examine two different scenarios that capture the fundamental processes of the secondary market. In subsection 4.1, I analyze the neutral price situation wherein the buyer and seller meet halfway between their valuations. The even split could arise due to balanced powers between buyers and sellers. In subsection 4.2, I analyze a process where buyers and sellers randomly match with each other and negotiate a resale price through pairwise bargaining. The matching and bargaining iteration goes on until at least one side of the market (buyer or seller) clears (Binmore and Herrero 1988; Rubinstein and Wolinsky 1990). This pricing mechanism is akin to consumers trading in an electronic marketplace such as Craigslist.

For each scenario, I first analyze traditional ticket selling and then contingency selling. I look for equilibrium where consumers' decisions are consistent with their beliefs. For explosion ease, I focus on the case of constrained capacity due to its practical and the analytical insight it provides. Unconstrained capacity can be analyzed in a straightforward manner as a special case where $SC \geq 2$.

4.1. Neutral Price

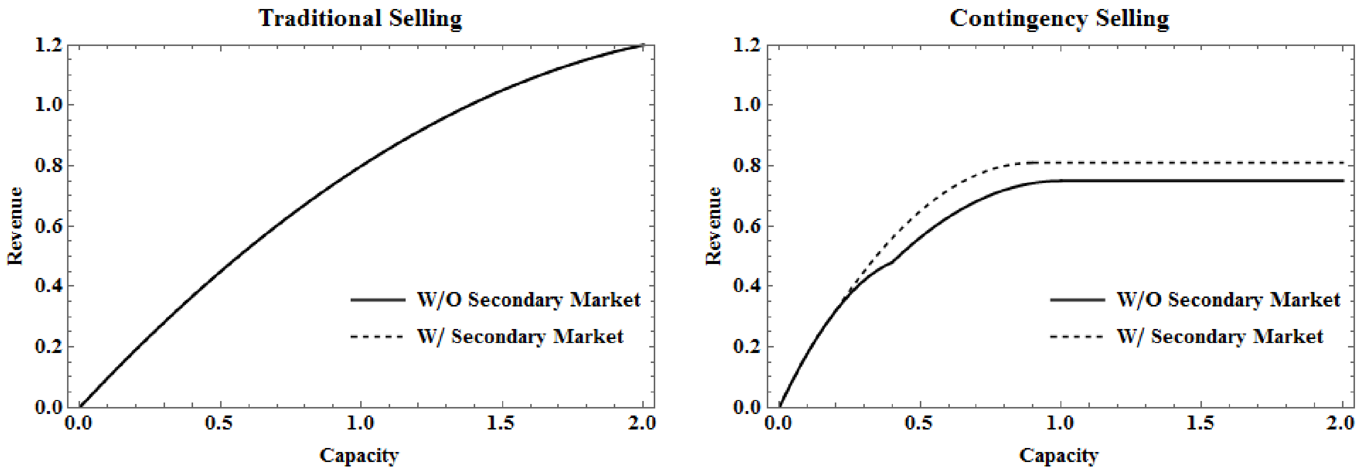
Under traditional selling, a potential buyer values a general ticket in the secondary market at 1 while a potential seller values it at β, regardless of the realized state. That means any price between β and 1 will clear the secondary market, making all potential buyers and sellers willing to trade at such a price. If the market clears so that buyers and sellers eventually obtain equal surplus from resale, the price will be $p = \frac{1+\beta}{2}$.[5] The surplus a buyer gains is $1 - \frac{1+\beta}{2} = \frac{1-\beta}{2}$, while a seller gains $\frac{1+\beta}{2} - \beta = \frac{1-\beta}{2}$.

Under contingency selling a ticket only has resale value in its corresponding state. For instance, if state B is realized, then A tickets have no resale value but a B ticket has the same resale value as a general ticket. Thus, the neutral resale price for a contingent ticket in its corresponding state is $\frac{1+\beta}{2}$. Tables 5 and 6 on page 65 outline the optimal prices and the corresponding sales for traditional ticket selling and contingency selling, respectively.

How does the existence of a secondary market impact the prices under the two selling mechanisms? Let me first look at traditional selling. Recall that the secondary market strengthens the motivations for both "buy now" and "buy later." For any given resale price p, the existence of the secondary market strengthens the "buy now" effect by increasing consumers' valuation for their less preferred state from β to p. At the same time, it strengthens the "buy later" effect by increasing the value of "buy later" from 0 to $1 - p$. With the neutral resale price, the resale price takes the middle value between β and 1, allowing the "buy now" and "buy later" effects to cancel out each other. Thus, for traditional selling, the optimal price is not influenced by the secondary market. This is illustrated in Figure 3, where the curves for firm revenues with and without the secondary market overlap in the left panel.

[5] Notice that this price is akin to a mediator, or a centralized platform, choosing the price p that maximizes the joint surplus, $(p - \beta)(1 - p)$, of potential buyers and sellers. The optimal price is $p = \frac{1+\beta}{2}$. Thus, the neutral price model captures the essence of a mediator and provides a useful benchmark that incorporates the dynamics in the secondary market before I model a more nuanced negotiation mechanism in the next subsection.

Figure 3. Impact of Secondary Market with Neutral Resale Price on Firm Revenue



Parameter Values: $\beta = 0.6$

In contrast, the right panel of Figure 3 shows the situation of contingency selling. When SC is low, the firm charges the same price ($P_A = P_B = 1 - SC$) as in the main model. All contingent tickets go to the consumer segment that values the corresponding state the most. If this turns out to be the true state, those consumers will keep their tickets. Otherwise, the tickets have no resale value. In either case, no resale occurs. Thus, with contingency selling, the secondary market does not influence firm strategy when SC is low.

When capacity is high, recall that the firm would sell contingent tickets for each state to both consumer segments. For instance, ticket A is priced such that both segments 1 and 2 consumers buy it. The price is determined by segment 2 consumers' willingness to pay because they are the low valuation segment. With a secondary market, segment 2 consumers have an additional motivation to buy simply because they can resell the tickets to segment 1 at a price higher than the value they would receive from watching the game in state A (i.e., $\frac{1+\beta}{2} > \beta$). This enhanced motivation to "buy now" for segment 2 consumers allows the firm to charge a higher price for contingent tickets relative to the main model. As a result, the revenue is also increased. Proposition 1.3 summarizes the comparison between traditional selling and contingency selling when the secondary market exists with neutral price:

Proposition 1.3. With neutral price being the resale price, contingency selling always generates higher revenue than traditional selling when $\beta < \sqrt{5} - 2$. When $\beta \geq \sqrt{5} - 2$, contingency selling generates higher revenue than traditional selling if $SC < \frac{4-\sqrt{2(-1+3\beta+5\beta^2+\beta^3)}}{4-4\beta}$.

Proposition 1.3 shows that the key result from the main model still holds in the presence of the secondary market. That is, contingency selling outperforms traditional selling when the capacity constraint is tight. Moreover, when β low, contingency selling is preferred at any capacity levels.

Note that when the secondary market exist, contingency selling outperforms traditional selling for a *larger* range of market conditions. Specifically, the threshold below which contingency selling is preferred is $SC = \frac{4-\sqrt{2(-1+3\beta+5\beta^2+\beta^3)}}{4-4\beta}$, which is higher than the threshold $\overline{SC}$ in the main model. The increased dominance of contingency selling vis-à-vis traditional selling comes from the respective impacts of the secondary market on the two selling mechanisms. As discussed earlier, when SC is low, the existence of the secondary market does not change the firm's revenue for either traditional selling or contingency selling. When SC is high, however, the secondary market improves the revenue for contingency selling but not for traditional selling. Taken together, the existence of secondary market enlarges the advantage of contingency selling.

4.2. Rubinstein Negotiation Price

We now analyze a different mechanism for how the resale price in the secondary market is determined. I model a resale process in which sellers post their tickets for sale and engage in price negotiation with interested buyers. Such a process is common in online trading platforms such as Craigslist and eBay.

Rubinstein infinite-period (round) bilateral bargaining is one of the most well-known models for the process and the equilibrium of buyer-seller negotiations (Gale 1987; Binmore and Herrero 1988; Rubinstein and Wolinsky 1990; Osborne and Rubinstein 1990). The game has two parts – matching and bargaining. Each round starts with a match between an individual on one side of the market (either buyer or seller) and at most another individual on the other. The probability of a given individual matching with someone is determined by the sizes of the buyer and seller markets. Let me denote B_t as the number of active buyers and S_t as the number of active sellers at the beginning of round t. If the numbers of participants on both sides are the same ($B_t = S_t$), then each participant will be able to match with someone from the other side. If the numbers of participants are different, say there are more sellers than buyers ($B_t < S_t$), then a seller can enter a match with probability $\frac{B_t}{S_t}$ and a buyer enters a match with probability 1. Similarly, the probability will be 1 for sellers and $\frac{S_t}{B_t}$ for buyers if there more buyers than sellers ($B_t > S_t$).

Bargaining then happens between each matched pair of buyer and seller. If they reach an agreement, the transaction will be conducted at the price level defined by Nash bargaining, and the pair leaves the secondary market. If no agreement is reached, both individuals remain active in the market for the next round. The game ends when either side has no individual remaining active in the game. Both buyers and sellers want to find a match and complete the trade sooner than later. Specifically, for every extra round that an individual stays active, her surplus will be discounted by $1-\gamma$, where $\gamma \in (0,1)$ represents consumer patience in the bargaining process. Larger γ implies more patient consumers.

The equilibrium is a combination of a price function $p(S,B)$ and value functions $V_s(S,B)$ and $V_B(S,B)$ for any $S,B \geq 0$. The price function assigns a price to each pair of matched buyer and seller given the active numbers of sellers (S) and buyers (B). The value functions define the maximum expected surplus for sellers and buyers respectively. In equilibrium, the price function will be such that all the matched sellers and buyers are willing to accept the equilibrium price whenever they are matched. The equilibrium price and valuations will depend on the bargaining power of the buyers versus the sellers, which in turn depends on the size of each segment. The equilibrium under this model formulation is summarized in Lemma 1.

Lemma 1. In equilibrium, all matched pairs reach an agreement in the first round of bargaining with an agreed resale price. The equilibrium resale price is $p = \frac{1+(1-\gamma)\beta}{2-\gamma}$ for $S < B$, $p = \frac{1+\beta}{2}$ for $S = B$, and $p = \frac{1-\gamma+\beta}{2-\gamma}$ for $S > B$.

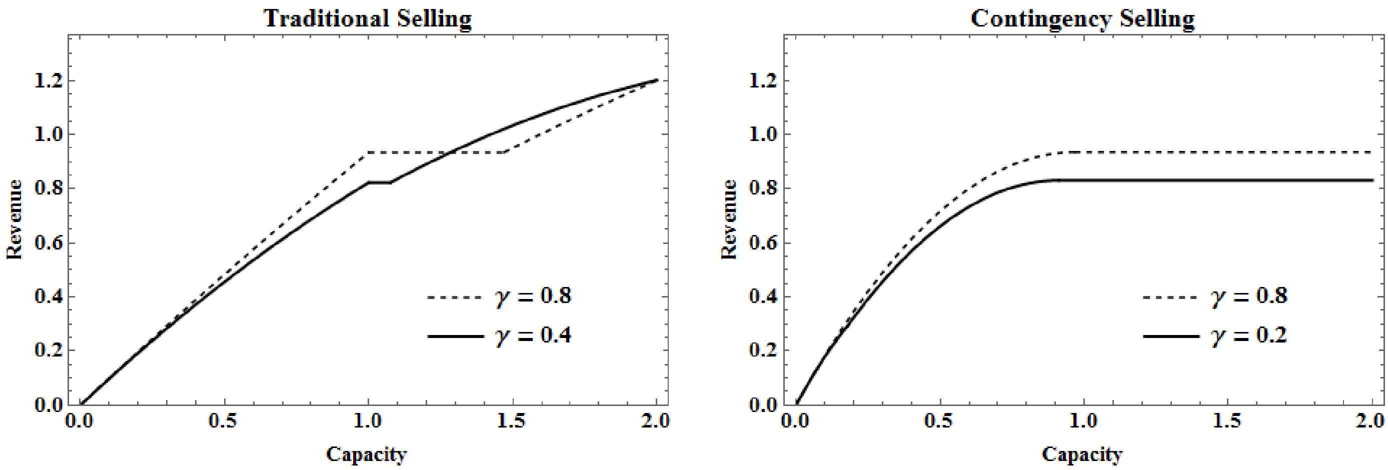
Lemma 1 is a modified version of the classic result in Osborne and Rubinstein (1990, Chapter 6). It shows that the resale price reached through bilateral bargaining depends on the relative bargaining power of buyers versus sellers. When there are equal numbers of sellers and buyers, the resale price is the neutral price that was analyzed in the previous section. When there are unequal numbers of sellers and buyers, the resale price depends on which side of the market has more participants: the price is lower (higher) if there are more (less) sellers than buyers. Since a higher resale price means more surplus for sellers and less surplus for buyers, Lemma 1 is consistent with the notion that the short side of the secondary market (i.e., the side with relatively fewer participants) has more power in the bargaining process than the long side.

Furthermore, the division of transaction surplus depends on consumers' patience (γ) in the bargaining process. When consumers are more patient (i.e., γ is higher), sellers and buyers are more willing to leave their matched partner and stay for extra rounds of matching and negotiation. However, not achieving an agreement has different consequences on the two sides of the market. An individual on the short side knows that she will have a high chance of finding a match in the next round of negotiations. On the contrary, an individual on the long side knows it can be very difficult for her to enter a new match if she leaves the current match. In other words, the long side of the market is simply more eager to find a match in the current round than the short side. Technically, higher patience benefits the short side more than it does the long side and leads to a more asymmetric division of surplus in favor of the short side.

Based on Lemma 1, I solve for the firm's optimal pricing strategy under traditional selling and contingency selling. Tables 7 and 8 on page 66-68 provide the results with proofs detailed in Appendices. Different from previous sections, consumer patience γ plays a role in determining the firm's prices and revenues.

We first discuss the effect of patience on the pricing. Under traditional selling, the impact of γ on revenue depends on the value of SC (illustrated in the left panel of Figure 4). When capacity is relatively low, $SC < 1$, only a limited number of consumers will get tickets in the first period. Thus, there will be more buyers than sellers in the secondary market. Since higher consumer patience increases resale price (p) when there are more buyers than sellers, the potentially higher resale price enables the firm to increase the price (P) in the first period and earn higher revenue. The opposite holds true when the capacity is high, i.e., $SC \geq 1$. A higher γ results in a lower resale price, which drives the firm to charge a lower price in period 1 to attract more buyers early and thus earn less revenue.

Figure 4. Impact of Consumer Patience on Firm Revenue



Parameter Values: $\beta = 0.6$

Under contingency selling (Table 8), similar to the case of neutral resale price, resale does not happen if capacity is sufficiently small, i.e., when $SC < \frac{1-\beta-\gamma+\beta\gamma}{2-\gamma}$. When SC is in the intermediate region ($\frac{1-\beta-\gamma+\beta\gamma}{2-\gamma} \leq SC < \frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$), the firm serves both segments and sells contingent tickets up to its capacity. When SC is high ($SC \geq \frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$), the firm's optimal strategy is not bounded by the capacity constraint. It sells as much as $\frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$ contingent tickets. Note that this amount is smaller than 1, so there will always be fewer sellers than buyers in the secondary market. Thus, higher consumer patience increases the resale prices of contingent tickets and in turn drives the firm to charge higher prices in the first period. To summarize, higher γ does not change the performance of contingency selling for low SC and increases it for high SC. These can be observed from the plots in the right panel of Figure 4. Proposition 1.4 summarizes the comparison between the two selling mechanisms.

Proposition 1.4. With bilateral bargaining and Rubinstein negotiation price in the secondary market, contingency selling always generates higher revenue than traditional selling if $\beta < \sqrt{5} - 2$ or

$\gamma \geq \hat{\gamma}$. If $\beta \geq \sqrt{5} - 2$ and $\gamma < \hat{\gamma}$, contingency selling (traditional selling) generates higher revenue if $SC < \widehat{SC}$ ($SC \geq \widehat{SC}$), where $\widehat{SC} = \frac{-2+(2-\beta)\gamma+\sqrt{\frac{1-\beta^3(1-\gamma)^3+\beta^2(-5+11\gamma-9\gamma^2+2\gamma^3)-\beta(3+7\gamma-15\gamma^2+5\gamma^3)+5\gamma-9\gamma^2+3\gamma^3}{-2+\gamma}}}{2(-1+\beta+\gamma-\beta\gamma)}$.

Proposition 1.4 shows that my key results continue to hold: contingency selling is advantageous when capacity is more limited (i.e., smaller SC), or consumer preferences are heterogeneous (i.e., smaller β).

How does consumer patience affect the benefit of contingency selling? The capacity threshold $\widehat{SC}$ increases in γ. That is, higher consumer patience makes it easier for contingency selling to outperform traditional selling. The threshold $\widehat{SC}$ is in the high SC region ($SC > 1$). Recall that, in this region, higher consumer patience increases the revenue in contingency selling but decreases that in traditional selling. Therefore, higher consumer patience makes it more likely that contingency selling dominates traditional selling.

In summary, the analysis in Section 4 shows that the core advantage of contingency selling over traditional selling holds in the presence of secondary markets and for different pricing mechanisms of the resale price. The secondary market generates an important trade-off for consumer motivations in terms of "buy now" versus "buy later." The existence of a secondary market does not necessarily improve the profitability of traditional selling because it enhances both motivations. Yet, contingency selling enables the firm to strike the balance better to improve profitability. As a result, the secondary market can enlarge the advantage of contingency selling over traditional selling.

5. Comparison with Other Selling Mechanisms

The focus of my paper is to examine the properties of contingency selling as a pricing model for advance selling. Thus, the most relevant alternative is the traditional selling method that sells general tickets in advance. The previous sections show that contingency selling outperforms traditional selling in some very common market conditions where the capacity is limited, and consumers have heterogeneous preferences.

In this section, I compare contingency selling with three other selling mechanisms, spot selling, advance selling with the possibility of full refund, and selling tickets as options, to further demonstrate its effectiveness. Since the main results of the paper get carried over to secondary markets, I exclude ticket resale from the analysis here.

In spot selling, the firm sells tickets after the true state is realized. If, for example, state A is realized, the firm can sell to segment 1 only (i.e., the high valuation segment) by charging a price $P = 1$, or to both segments by pricing at β. When SC is smaller than 1, the firm's best targeting strategy is always to target segment 1 since it is impossible to serve both. The firm can sell up to its full capacity SC for a revenue of SC. When $SC \geq 1$, however, the firm has two options. If it sells to segment 1 by pricing at 1, it will sell one ticket and make 1 in revenue. If it prices the ticket at β and sells to both segments up to capacity SC, the revenue will be $SC\beta$. Table 9 on page 69 summarizes the firm's optimal price and revenue under the two states.

Proposition 1.5 summarizes the comparison of revenues based on Table 2b and Table 9:

Proposition 1.5. When $\beta < \frac{1}{2}$, contingency selling (spot selling) generates higher revenue if $SC < \frac{1}{2}$ ($SC \geq \frac{1}{2}$). When $\beta \geq \frac{1}{2}$, contingency selling (spot selling) generates higher revenue if $SC < \frac{3}{2} - \frac{1}{2\beta}$ ($SC \geq \frac{3}{2} - \frac{1}{2\beta}$).

Contingency selling outperforms spot selling with low SC, even though both selling mechanisms enable the firm to conduct inter-state price discrimination, i.e., tailoring the price of each state to its corresponding high valuation segment. What makes contingency selling superior is that it also enables the firm to take advantage of consumer heterogeneity in beliefs (captured by parameter ρ) in addition to the heterogeneity in preferences. When the firm sells tickets on the spot, the uncertainty has already been resolved. Thus, all consumers within each segment will have the same valuation for the ticket. With low SC, the firm charges $P = 1$ and sells the tickets to fans whose preferred state is realized. Such price generates a margin of 1. When the firm sells contingent tickets in advance, however, it is unclear which team will be playing in the game. Due to heterogeneous beliefs, consumers assess a contingent ticket at different values: a more optimistic consumer would value a contingent ticket higher than a pessimistic consumer. Thus, through *belief-based price discrimination*, contingency selling enables the firm to charge a price that only optimistic consumers are willing to pay. For both A and B tickets, the firm charges a price greater than $\frac{1}{2}$, and the total margin is higher than 1.

Proposition 1.5 also shows that the capacity threshold under which contingency selling outperforms spot selling is (weakly) increasing in β. The underlying reason is the different effects of β on the two selling mechanisms. In contingency selling, higher β enables the firm to sell contingent tickets to serve consumers of both segments. The higher β, the higher price the firm charges. However, under spot selling, the value of β has no impact on revenue as long as $SC \leq 1$ since the firm only sells to the high valuation segment. Thus higher β enlarges the advantage of contingency selling over spot selling.

Next, I examine advance selling with the possibility for full refund. Under this mechanism, the firm sells tickets in advance with uncertainty but, after the uncertainty is resolved, a consumer who has bought a ticket can choose to opt out and get fully refunded. In this sense, all consumers are willing to purchase the ticket so long as the price matches their valuation for their preferred state. When $P \leq \beta$, all the consumers are willing to purchase the ticket and will not opt for refund. When $\beta < P \leq 1$, all the consumers are willing to purchase the ticket but those whose preferred state is not realized will opt out to be refunded. Therefore, the optimal price for the firm to charge is either β or 1. The demand is 2 if $P = \beta$ and 1 if $P = 1$. Note that this demand function is the same as that under spot selling. Since some consumers will eventually opt out under ticketing with refund, the revenue cannot be higher than that in spot selling revenue. Thus, contingency selling's advantage over spot selling continues to hold for advance selling with the possibility for full refund.

Finally, I consider consumer option and employ the specific mechanism proposed by Sainam et al. (2010) to model option pricing. Specifically, the firm sets two prices, an option price P_O and an exercise price P_E. The option is offered when the true state is uncertain and consumers can purchase the option by paying the option price. After the true state is realized, consumers who hold the option can exercise the option by paying the exercise price. One immediate observation is that $P_O + P_E$ cannot be greater than 1. Otherwise, no consumer will purchase the option in the first stage because the total cost of watching the game exceeds their valuation even in their preferred state. Thus, the margin per seat cannot be higher than 1. This fact leads to the conclusion that, when the capacity constraint is binding, the revenue under consumer option cannot exceed that under spot selling. Therefore, the contingency selling's advantage over spot selling still applies to consumer option.

6. Concluding Remarks

In this paper, I propose the idea of contingency selling in the context of sports markets where consumers face uncertainty about the product and there is service capacity constraint. I examine its properties in comparison with the traditional approach of selling tickets and show that contingency selling is able to extract (higher) rent for each seat through interstate price discrimination. By decoupling different states, contingency selling enables the firm to sell each state to the right kind of consumers, i.e., the consumers who have a stronger preference for a particular team and/or have a stronger belief in the team's chance of advancing to the game. Contingent tickets essentially enable the firm to sell every single seat to multiple consumers. This effect becomes more pronounced when service capacity is more limited.

In terms of theoretical contribution, to the best of my knowledge, this is the first study that formally examines the merits (and limitation) of contingency selling, an idea that has been experimented with in the industry. my model integrates the literature on advance selling and ticket pricing by considering the impacts of product uncertainty, capacity constraints, and consumer heterogeneity, as well as secondary markets and the different price mechanisms for resale.

Our model provides valuable guidance on the nascent practice of contingency selling for sports leagues and event organizers. my study points out the revenue benefit of viewing each seat in a venue as generating different values for different consumers, depending on the alignment between a consumer's preference for teams and which team is playing in the game. The contingent ticket approach that I propose can decouple different values so the right consumer purchases the seat for the right event, revenue can be enhanced. This gain becomes larger when the venue has a service capacity constraint because the opportunity to match the seat with the right consumer becomes more critical.

In practice, the contingency selling model I proposed is straightforward to implement. The firm needs to know consumers' preferences to watch their favorite teams play (V) and their preference discount to watch other teams (β). As suggested in Sainam et al. (2010), these estimates can be gleaned by hosting a Vickrey-type auction (second-price sealed-bid auction) among representative consumers. Alternatively, fans can be surveyed directly for their preferences.

Note that V is required to implement either traditional selling or contingency selling. However, while the firm must know β to effectively practice traditional selling, it only needs β for contingency selling when service capacity is high. When capacity is limited, which is the situation for contingency selling to be more effective than traditional selling, the firm does not need to estimate β to implement contingency selling. This is because contingency selling enables the firm to price different states separately and when capacity is low, the firm prices each state to accommodate only one consumer segment. Knowing consumers' preferences for their favorite teams is sufficient. Thus, implementing contingency selling does not burden the firm with additional information acquisition.

We end the paper with two potential venues for future research. First, in modeling the secondary market, I focus on the neutral price scenario, which can be achieved by equal-powered buyers and sellers, and the bilateral negotiation scenario, which is common in marketplaces such as Craigslist. Another scenario to consider is when the firm can actively participate in the secondary market. It is then possible to explore how contingency selling may change with dynamic pricing, that is, the firm setting one price in advance selling and another in the spot market.

Second, it will be interesting to further examine the properties of contingency pricing when more nuanced consumer situations are assumed. For instance, fans may be risk-averse when making decisions under uncertainty (Kahneman and Tversky 1979), or fans of different teams may have different amounts of enthusiasm for watching their favorite teams playing. These situations can be captured by enriching the consumer characteristics assumptions in this model.

Essay 2. Marketing Mix in Reward-based Crowdfunding

1. Introduction

"I could make a much cheaper work, but part of the appeal of Kickstarter and crowdfunded projects is to make something of higher quality than I could without having the capital there at the beginning."[6]

-----Wolfgang Baur, founder of over 25 crowdfunding projects

In recent years, reward-based crowdfunding through platforms such as Kickstarter and Indiegogo has become an increasingly popular approach for entrepreneurs to launch new products.[7] In 2018 the worldwide transaction value in reward-based crowdfunding had amounted to 9.4 billion US dollars and is expected to continue growing at an annual rate of 29%.[8] Through reward-based crowdfunding, entrepreneurs seek funding to support the production of a wide variety of products such as postcards, handbags, electronic devices, household items, music albums, movies, and so on. The mechanics of how a reward based crowdfunding platform works can be seen from a simple illustrative example of a Kickstarter-based crowdfunding campaign (see Figure 5 below). To start a crowdfunding campaign, entrepreneurs need to post information such as the description of the project (including price), the funding goal, the funding period, and the reward for the funders through platforms such as Kickstarter.com. Typically, the reward takes the form of the product funders invest in. The project information posted by entrepreneurs is visible to funders who are registered on the crowdfunding platform. Based on the information they see, funders decide whether to fund a project or not. When funders pledge support to the project they authorize the payment in case the funding goal is met. However, the payment is not actually rendered until the project goal is met.

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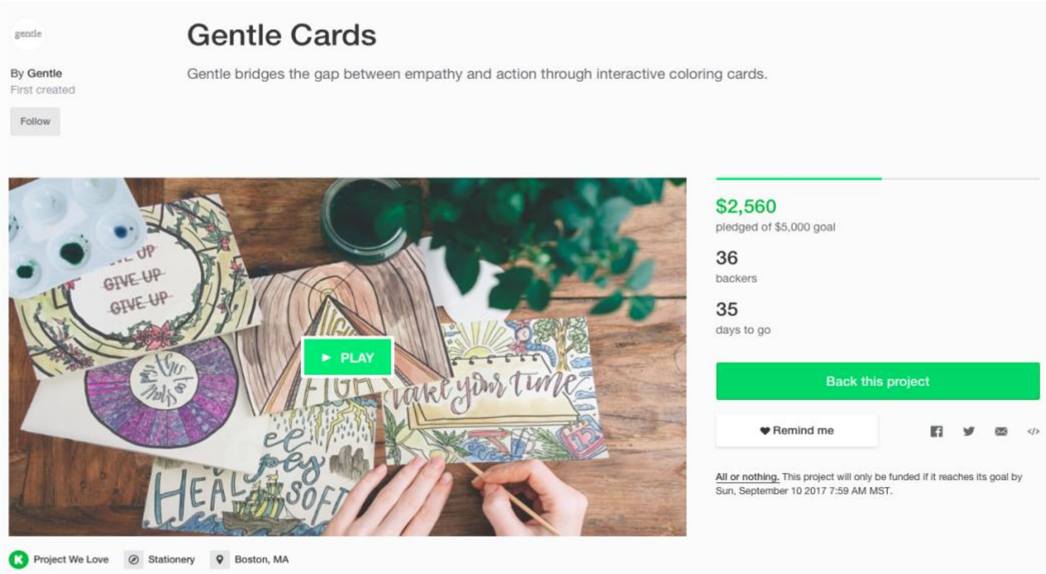
[6] For source, see https://www.backerkit.com/blog/stretch-goals-pros-and-cons/, accessed on 09/18/2018.

[7] The other types of crowdfunding include donation-based, lending-based, and equity-based crowdfunding. Compared with these other types, award-based crowdfunding is unique in that it directly allows the entrepreneur to promote a new product idea and seek funding to support the production of the new product. See Burtch et al. 2013; Cumming and Johan 2013; Mollick 2014 for more details on different crowdfunding types. my paper focuses on reward-based crowdfunding.

[8] For source, see https://www.statista.com/outlook/335/100/crowdfunding/worldwide, accessed on 07/18/2018.

Figure 5. The project "Gentle Cards" on Kickstarter



Therefore, if within the funding period, the total amount of money raised for a project reaches its funding goal, funders' payment gets rendered and the entrepreneur gets all the funds to start producing the product. The funders will receive their rewards once the products get produced. On the other hand, if the funding goal is not met, the funders do not get charged. Thus, crowdfunding follows the "All or Nothing" (AON) model, which is the dominant approach to crowdfunding (Mollick 2014).[9] Under crowdfunding, a funder (buyer) pays for the new product before it is produced.

Given the mechanics of the crowdfunding an important distinction between the crowdfunding model and the traditional approach of market entry is worth noting. With traditional market entry, entrepreneurs or firms rely on debt or equity financing to fund the business operations. Prior to that they rely on their judgment or use market research to access market potential, make requisite investment, produce the product and then sell it to potential consumers. More specifically, with traditional market entry, the entrepreneur often time relies on sales forecast generated from market research (Neelamegham and Chintagunta 1999). However, market research can often times be unreliable for multiple reasons. Due to issues such as the limitation in sampling and the reliance on consumer perception rather than actual purchase, does not always provide an accurate picture of the actual

[9] The other approach is called "Keeping it All", under which the fundraiser keeps all the money she/he collects regardless whether it reaches the funding goal or not.

demand.[10] Moreover, prior research has shown that historical data and past performance, often utilized in market research, can be poor predictors of market conditions post launch (Hitsch 2006, Luan and Sudhir 2010). On the contrary, in crowdfunding, due to the AON feature demand is demonstrated before the entrepreneur enters the production cycle. For this reason, crowdfunding has the potential to mitigate the impact of market uncertainty. In other words, because of the AON feature, market uncertainty is resolved ex-ante. By seeking funding support from the crowd, crowdfunding taps into actual consumer purchase through the reward offered in exchange for funding before a project starts. As a result, crowdfunding has the potential to protect the entrepreneur against the situation where the demand for the new product is low.

In this paper using a model where a monopolist entrepreneur facing a market with uncertain demand. I hone in on the AON feature to analyze marketing strategies associated with reward-based crowdfunding. In particular, this paper answers the following research questions. How does the entrepreneur or firm behave differently in product quality and pricing decisions if it enters the market through crowdfunding rather than the traditional approach? What are the implications for advertising that firms utilize to inform potential consumers? What are some of the welfare implications of using reward-based crowdfunding platforms? How does the degree of market uncertainty affect these decisions? Answers to these questions not only enrich my understanding of crowdfunding for managerial purposes, but also generate useful insights for consumer welfare and public policy regarding crowdfunding as a new business model. It also complements the existing literature on crowdfunding and decision making in markets with uncertain demand.

The key insights from my analysis are as follows. First, quite intuitively, the all-or-nothing feature of crowdfunding platform safeguards the entrepreneur against poor market conditions: if there are not enough consumers who like the product, the funding target cannot be reached, and the entrepreneur does not receive any funding nor produce anything. This safeguarding effect is a direct benefit of crowdfunding. Second, because of the direct benefit, the entrepreneur in crowdfunding is able

[10] For the source, see https://smallbusiness.chron.com/disadvantages-market-research-new-product-development-23441.html, accessed on 07/19/2018.

to advertise more which directly boosts the effective demand that the entrepreneur faces. Moreover, the demand safeguards increases the equilibrium product quality. In a market with heterogeneous preference for quality, the entrepreneur is also able to charge a higher price in equilibrium. Increase in quality has a dual effect. On one hand, increase in quality may decrease the probability that the crowdfunding project will be successful. That happens because, a higher quality product costs commensurately more to produce. As a result, the funding goal required for the project to be successful also increases. Higher funding goal decreases the probability of a success. However, on the other hand, increase in quality also ensures that if the project is successful, the entrepreneur will be able to extract a higher revenue from a secured group of consumers (funders). Ex–ante due to the demand safeguard the latter effect prevails and the entrepreneur is able to make higher profits in equilibrium. Equilibrium profit is also boosted by increased advertising in equilibrium. Increase in profit due to higher levels of advertising, quality and price is the indirect benefit due of safeguarding. Both the direct and indirect benefits come from the unique feature of crowdfunding and provide important implications for entrepreneurs and policymakers. I test the robustness of my core findings through extensions. In the first extension I outline the effect of marginal changes in overall uncertainty in the market. my main finding in this section is – increased uncertainty also increases the optimality of crowdfunding platform because the entrepreneur makes higher profit as uncertainty increases. In other words, the efficacy of crowdfunding vis-à-vis traditional method of selling increases in a more volatile market. In a more volatile market, demand is more on the extremes – very low or very high. Demand safeguards associated with crowdfunding allows the firm to focus more on high demand which motivates the entrepreneur to increase advertising and quality provision in equilibrium. Expected profit also increases as a result. In another extension I allow the possibility that consumers' willingness to pay in crowdfunding and traditional markets are different. While confirming core findings of the paper, I also show that an entrepreneur can expect to make higher profit through a crowdfunded market even when the average willingness to pay for a product bought from the crowdfunding platform is lower than willingness to pay for a product acquired through traditional channel. In other words, my results suggest that under proper market conditions, the benefits of crowdfunding prevail even when traditional market entry possess ex-ante advantages in turn lending support to my core results.

The remainder of this essay is organized as follows. Section 2 provides a review of related literature in marketing, economics, and entrepreneurship. In Section 3, I develop the base model for the crowdfunding context. Section 4 compares an entrepreneur's product and pricing strategies in crowdfunding vs. the traditional entry scenario. Section 5 generalizes the model to count for the role of informative advertising as well as welfare implication. In Section 6 provide some viable extension of the core model. Finally, I conclude the paper with summary and implications. The Appendices provide necessary technical details.

2. Related Literature

An emerging literature highlight the multitude of benefits accrued by firms choosing to launch and sell products on a crowdfunding platform. For example, two recent papers argue that crowdfunding provides firms the possibility of price discrimination. Belleflamme et al. (2013) point out that one key value of reward-based crowdfunding is that it enables the entrepreneur to price discriminate between people who pledge early and pledge late. More closely to this work is a recent work by Hu et al. (2015). They examine entrepreneurs' marketing strategy - pricing and product line - in the crowdfunding context. They find that crowdfunding enables entrepreneurs to engage in price discrimination by offering a menu of prices. The high valuation buyers are willing to pay the higher price on the menu to increase the project's chance of meeting the funding goal even if a lower price is also available on the menu. They further analyze the entrepreneur's product line decision and find that crowdfunding leads to less differentiation within the product line. In this paper, the entrepreneur's benefit from crowdfunding persists without price discrimination. The primary source of advantage derives from safeguarding benefits of the AON feature associated with crowdfunding which allows the entrepreneur to provide a higher quality product and advertise more.

A key benefit of crowdfunding is that crowdfunding provides information about market conditions. Chemla and Tinn (2018) show that crowdfunding enables firms to learn about consumer preferences based on the contribution of the crowd. This learning benefits the firms regardless of whether they achieve the funding goal. Mollick and Nanda (2016) validate the revelation value of crowdfunding empirically with Kickstarter data. Viotto da Cruz (2018) find that, among the projects that fail to meet their funding goals, those who receive more positive feedback are more likely to be released in the market. Roma et al. (2018) show that entrepreneurs can utilize the success of crowdfunding campaign as a signal of good market potential to convince venture capitalists to invest in their projects. However, none of the above papers examine how crowdfunding's value in revealing market conditions impacts marketing strategies (e.g., product quality, price, and advertising), which is the primary objective of this paper. Therefore, this paper complements the existing literature by delineating the spillover effect on crowdfunding on some of the marketing mix elements.

This paper is fundamentally about the mechanism through which crowdfunding deals with uncertain demand. Therefore, this work is related to the literature on marketing decisions under uncertainty. Demand uncertainty is a key topic in the literature of marketing and economics (Horowitz 1970, Dehez and Jacquemin 1975, Harris and Raviv 1981, Jagpal and Brick 1982, Carlton and Dana 2008). The effort at developing marketing tools to resolve demand uncertainty at least goes back to Fourt and Woodlock (1960). On the one hand, various market forecasting tools have been developed over time to predict the market response to new products (Bass 1969, Urban and Katz 1983, Hitsch 2006, Luan and Sudhir 2010). These techniques, although useful in many situations, are limited in eliminating demand uncertainty completely. The application of these techniques relies crucially on the availability of sales data of similar or analogous products. For new products with innovative features, such data are often hard to get. It is often inevitable for entrepreneurs and new startups to live and deal with uncertainty.

Some studies have analyzed different mechanisms that help firms to deal with demand uncertainty. Biyalogorsky and Koenigsberg (2014) examine the scenario where a firm introduces a sequence of new products. They show that when the degree of uncertainty is high, the firm will introduce a low-quality product initially and postpone the quality decision of the second product until the market condition is realized. Similarly, Che et al. (2010) point out that backorder, another form of postponement, can be adopted to tackle high demand uncertainty. Outlining the immediate benefits from crowdfunding to deal with market level uncertainty, in this paper, I dig deeper by parametrizing the degree of uncertainty and providing insights in to how marginal changes in the degree of uncertainty impacts the entrepreneur's profit in traditional vis-à-vis crowdfunded markets. my results suggest that increased uncertainty benefits firms. Akin to this result, Alexandrov (2015) also demonstrates that greater uncertainty (risk) may result in higher firm profit. However, the higher profit occurs only when the firm is able to adjust its marketing decisions (e.g., price) after the uncertainty is resolved. Syam et al. (2016) show a similar result in the sales management context; both the sales manager and the salesperson can benefit from higher sales uncertainty when the salesperson is able to acquire information about the realized market condition and adapt his effort based on such information. However, the results of both studies rely on the assumption that decision-makers are able to adjust their decisions after the uncertainty is resolved. my analysis shows that even without the possibility of adjusting marketing mix after uncertainty is resolved, the firm, in a crowdfunding market, may still derive higher profit as uncertainty increases.

3. Model Setup

We start by analyzing quality and pricing strategies in crowdfunding versus traditional market entry. I then model the advertising decision and its interaction with other marketing variables. Finally, I examine crowdfunding's implication on consumer (funder) surplus.

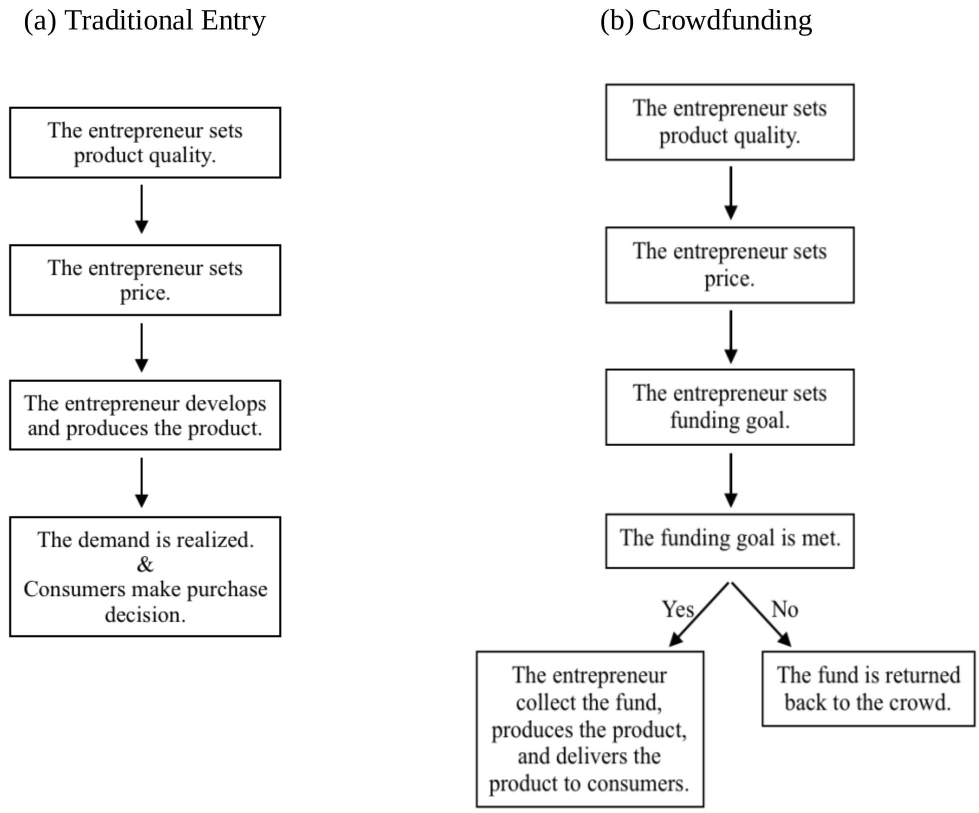
Our model formulation is akin to Carlton and Dana (2008). The market consists of a mass of M consumers. Each consumer is represented by her distinctive quality preference θ that follows the uniform distribution over $[0,1]$. For a consumer i, her willingness to pay for a product of quality S is $\theta_i S$. If I denote the price of the product as P, a consumer i's net utility from purchasing the product is $\theta_i S - P$. A consumer's utility is zero if she does not purchase the product. For any given quality-price pair (S, P), consumers with $\theta \geq \frac{P}{S}$ will purchase the product. In other words, the entrepreneur faces a downward slope demand function $D(P) = M\left(1 - \frac{P}{S}\right)$.

To capture the market uncertainty, I assume the market size M, which represents the market potential of the entrepreneur's project, is unknown to the entrepreneur before he or she enters the market. The entrepreneur only knows that M follows a uniform distribution over the range $[0,1]$. If the entrepreneur launches the production, she invests $C(S)$ to deliver a product of quality S. I assume a convex cost of quality provision, $C(S) = \frac{S^2}{2}$ (Mussa and Rosen 1978). Following the literature, I assume that the entrepreneur chooses the quality first and then the price (Moorthy 1984, Villas-Boas 1998, Lahiri and Dey 2013).

There are two critical assumptions that are worth noting. First, the distribution of consumer preference θ is the same in the traditional and the crowdfunding markets. Such an assumption may seem tenuous since one can claim that consumers with different preferences visit the two retailing platforms and therefore the distribution assumption on θ should be different as well. Second, one may also contend that the market size M that an entrepreneur expects to face is different in a traditional and crowdfunding markets. Indeed, it may be the case that potential mass of customers is larger in a traditional retailing platform. In the extension section 6 I address both the assumptions and show that the core results of the model hold given boundary market conditions.

The sequence of the game in both the traditional and crowdfunding market entry is outlined in Figures 6(a) and 6(b). In the traditional market entry, the entrepreneur decides on the quality of the product in Stage 1. In Stage 2, given quality, the entrepreneur decides on the price. Finally, in Stage 3, consumers make buying decision given the price and quality. A key difference between the two market-entry models is the time when the entrepreneur undertakes the cost to initiate production. In case of crowdfunding, entrepreneur takes action to produce only when the funding goal is met. Such is the AON feature that provides a safeguard to the entrepreneur. This is not true for the traditional way of starting a business—even though one can use market research to gauge market demand, actual reactions from real buyers will not be observed until the product is produced and sold through the retail channel.

Figure 6. Move Sequence in the Two Entry Models



Given the demand function $D(P) = M\left(1 - \frac{P}{S}\right)$, the optimal price given quality S is $P^* = \frac{S}{2}$ in both traditional and crowdfunding scenarios. In the following discussion, I outline the entrepreneur's equilibrium quality and pricing decisions under the two entry models – traditional and crowdfunding. I start with a simpler model with only quality and price decision (and no informative ads) in section 4. That allows me to delineate the two part effect of crowdfunding more clearly. Subsequently, in section 5 I consider the full model which reflects the core effects but also adds nuances that comes through endogenizing advertising.

4. Analysis

4.1. Benchmark Result with Fixed Quality

In order to understand optimal marketing strategy in the traditional and crowdfunding models, I first look at the entrepreneur's price decisions and expected profits for a fixed quality level $\bar{S}$. The reason why it is useful to first analyze the market with fixed $\bar{S}$ is because the indirect effect from crowdfunding emanates from endogenizing quality. Therefore, fixed quality allows me to quantify the direct effect.

Suppose that the product quality is $\bar{S}$ in both the traditional entry scenario and crowdfunding. In the traditional entry scenario, the entrepreneur chooses price P_T and incurs the production cost $\frac{\bar{S}^2}{2}$ before the uncertainty is resolved. Depending on the realized market size M, her profit is $\pi_T(\bar{S}, P_T) = M\left(1 - \frac{P_T}{\bar{S}}\right)P_T - \frac{\bar{S}^2}{2}$. Given the aforementioned equilibrium price $P_T^* = \frac{\bar{S}}{2}$, I can rewrite the profit function as $\pi_T^*(\bar{S}) = \frac{M\bar{S}}{4} - \frac{\bar{S}^2}{2}$. Since M follows the uniform distribution over $[0,1]$, the entrepreneur's expected profit is $E[\pi_T^*(\bar{S})] = \frac{\bar{S}}{8} - \frac{\bar{S}^2}{2}$.

In the crowdfunding scenario, the entrepreneur launches the production only if the amount of raised funds meets the funding goal. If the funding goal is not reached, funders get their money back, and the entrepreneur does not enter the market. Thus, her profit is $M\left(1 - \frac{P_C}{\bar{S}}\right)P_C - \frac{\bar{S}^2}{2}$ if $M\left(1 - \frac{P_C}{\bar{S}}\right)P_C \geq G$ and zero otherwise. By substituting the optimal price $P_C^* = \frac{\bar{S}}{2}$, I can summarize the entrepreneur's profit as follows:

$$\pi_C(\bar{S}, P_C, G) = \begin{cases} \frac{1}{4}M\bar{S} - \frac{\bar{S}^2}{2}, \frac{1}{4}M\bar{S} \geq G \\ 0, \frac{1}{4}M\bar{S} < G \end{cases} \tag{4}$$

Since M follows $[0,1]$ uniform distribution, the probability for $\frac{M\bar{S}}{4} \geq G$ to be true is $1 - \frac{4G}{\bar{S}}$. The entrepreneur's expected profit can be written as:

$$\pi_C[\pi_C(\bar{S}, P_C, G)] = \left(1 - \frac{4G}{\bar{S}}\right)\int_{\frac{4G}{\bar{S}}}^{1}\left(\frac{1}{4}M\bar{S} - \frac{\bar{S}^2}{2}\right)dM = \frac{1}{2}\left(1 - \frac{4G}{\bar{S}}\right)\left(4G + \bar{S} - \frac{\bar{S}^2}{4}\right) \tag{5}$$

We take the first order derivative of the profit expectation with respect to the funding goal $\frac{dE[\pi_C(\bar{S}, P_C, G)]}{dG} = 2\bar{S} - \frac{4G}{\bar{S}}$ and obtain $G^* = \frac{\bar{S}^2}{2}$. I observe that the optimal funding goal equals to the cost of production. The intuition is: G represents the minimum funding needed for the crowdfunding campaign to succeed. The lower G is, the larger the chance that the crowdfunding campaign is successful. The entrepreneur wants to increase the chance as long as she makes a non-negative profit. That means the entrepreneur wants to set G as low as possible given that she can break even. The minimal income that the entrepreneur needed to break even is the production cost $\frac{\bar{S}^2}{2}$. Thus, the optimal funding goal G is the production cost $\frac{\bar{S}^2}{2}$.

Substituting $G^* = \frac{\bar{S}^2}{2}$, the profit expectation in crowdfunding market is:

$$E[\pi_C^*(\bar{S})] = \frac{\bar{S}}{8} - \frac{\bar{S}^2}{2} + \frac{\bar{S}^3}{2} \tag{6}$$

Simply put, safeguarding allows the entrepreneur to earn an extra profit of $\frac{\bar{S}^3}{2}$ in the crowdfunding market ($E[\pi_C^*(\bar{S})] - E[\pi_T^*(\bar{S})] = \frac{\bar{S}^3}{2}$). The term $\frac{\bar{S}^3}{2}$ captures the enhanced profit due to safeguarding. Since it manifests for any given quality level, I call this the *direct* effect of safeguarding in crowdfunding. Moreover, the direct effect of safeguarding is increasing in the product quality $\bar{S}$. Higher quality is associated with a higher cost. When the cost is higher, the entrepreneur is more worried about wasting such cost in a poor market condition (small demand). In turn, safeguarding is more valuable. Next, I consider the entrepreneur's optimal strategy with endogenous quality.

4.2. Endogenous Quality

If the entrepreneur enters the market through the traditional channel, she chooses a quality level S_T to maximize the expected profit. $E[\pi_T(S_T)] = \frac{S_T}{8} - \frac{{S_T}^2}{2}$. It turns out that the profit-maximizing quality is $S_T^* = \frac{1}{8}$. The expected profit becomes $\frac{1}{128}$. If, instead, the entrepreneur launches her product through crowdfunding, she sets quality S_C to maximize the expected profit $E[\pi_C(S_C)] = \frac{S_C}{8} - \frac{{S_C}^2}{2} + \frac{{S_C}^3}{2}$. It can be verified that the optimal quality is $S_C^* = \frac{1}{6}$. The entrepreneur earns a profit expectation of $\frac{1}{108}$. This result is summarized in the following proposition, whose proof is presented in Appendix 15.

Proposition 2.1. The product quality, as well as the price, is higher if the entrepreneur enters the market through crowdfunding rather than through the traditional approach. The expected profit is also higher through crowdfunding.

Proposition 2.1 allows me to see the second *(indirect*) effect of crowdfunding on the entrepreneur. The entrepreneur's expected profit is $E[\pi_T(S_T)] = \frac{S_T}{8} - \frac{{S_T}^2}{2}$ in the traditional market entry scenario and $E[\pi_C(S_C)] = \frac{S_C}{8} - \frac{{S_C}^2}{2} + \frac{{S_C}^3}{2}$ in crowdfunding. Apart from the direct safeguarding effect measured by the last term of $E[\pi_C(S_C)]$, the indirect effect of crowdfunding is realized because $S_C^* > S_T^*$, which also makes the total of the first two terms in $E[\pi_C]$ larger than $E[\pi_T]$. Thus, the entrepreneur benefits from crowdfunding through the enhanced product quality in addition to the direct effect through safeguarding.

The intuition behind this second-tier benefit lies in greater quality provision in a crowdfunded market. A higher quality product allows the entrepreneur to charge a higher price and extract more revenue from the market. A caveat however is in order. Choosing a higher level of quality comes with two countervailing forces. On one hand, higher quality enables the entrepreneur to earn a higher per-unit margin. On the other hand, delivering higher quality is also commensurately costlier because of the convex cost of quality. However, demand safeguarding ensures that the former effect prevails. To see this, notice in crowdfunding, higher quality implies that the ex-ante probability of successfully meeting the funding goal is lower. This is because, in equilibrium, the probability of a successful project is $\Pr(M > 2S_C) = (1 - 2S_C)$ which is a decreasing function of S_C. However, the all-or-nothing feature of crowdfunding safeguards the entrepreneur against poor market conditions (i.e., small market size). Thus, the entrepreneur is relatively less concerned about the situation where the market size is too small to cover the production cost. Instead, the entrepreneur can focus on the market size conditional on the project being successful which is given by $E[M|M > 2S_C] = \frac{1}{2} + S_C$ and it is increasing in S_C. This increases the incentive to optimally increase quality. Higher quality enables the entrepreneur to charge a higher price in crowdfunding because equilibrium price takes the form of $\frac{S_C}{2}$. Taken together, with a higher quality product sold at a higher price the entrepreneur derives higher profit. On the other hand, in the traditional market, the entrepreneur makes the quality decision based on the expected market size. In the absence of demand safeguards, the entrepreneur considers the convex cost of quality which in turn forces the entrepreneur to choose a lower level of quality. Given the above dynamic outlining the rationale behind higher quality provision with crowdfunding, I move on to the full model considering entrepreneur's' optimal advertising decision along with the other marketing mix variables.

5. Full Model with Informative Advertising

One implicit assumption so far is that all the consumers in the market are aware of the new product. Yet, in neither traditional market entry nor crowdfunding will consumers automatically become informed of new products. Often, entrepreneurs need to inform consumers of new products through advertising. In fact, many successful crowdfunding projects are accompanied by well-crafted marketing campaigns, often with the help from professional agencies (e.g., Enventys Partners).[11] As Roy Morejon, president of Enventys Partners, once stated, "Running a crowdfunding project is not the sort of thing where you set it up, launch, and then come back 30 days later to collect your money."[12] Such marketing campaigns usually take the form of promotion on social media and happen prior to the launch of crowdfunding projects. It is generally recommended by industry experts that promotions take place 1-3 months in advance of the launch of a crowdfunding project.[13] In this section, I extend the base model to analyze the entrepreneur's equilibrium advertising strategy by modeling promotions as entrepreneurs' posting informative ads to raise awareness of its products (Grossman and Shapiro 1984). Of particular importance is to outline how equilibrium advertising strategy interacts with equilibrium quality decisions.

To account for the advertising decision, I add an additional stage (Stage 0) to the beginning of the model. Initially, no consumer is aware of the new product. Consumers become aware of the new project via the entrepreneur's informative advertising. To inform $K \in [0,1]$ portion of the market population, the entrepreneur needs to spend expenditure $ad(K) = \frac{K^3}{24}$ on advertising. I chose this particular functional form for two reasons. First, it guarantees the existence of closed form solutions. Second, it is consistent with the literature on informative advertising where the cost of advertising is

[11] Zack Miller, 2018, "A List of the Best PR Firms for Crowdfunding," https://www.thebalancesmb.com/a-list-of-the-best-pr-firms-for-crowdfunding-985173, accessed on 08/29/2018.
[12] Gillian Terzis, 2017, "Do Paid Ads in Crowdfunding Pay?" https://www.backerkit.com/blog/does-paid-advertising-in-crowdfunding-pay, accessed on 08/29/2018.
[13] For the source, see https://enventyspartners.com/blog/facebook-advertising-and-your-crowdfunding-project/, accessed on 08/29/2018.

assumed to be continuous, increasing, and convex. The last assumption reflects the notion that it is increasingly expensive to reach additional consumers as the informed population gets larger.

In the following I focus on the firms advertising decision primarily because advertising is the added variable in this formulation. The dynamics behind equilibrium quality provision and price remain the same. Moreover, to better understand the equilibrium advertising strategy, akin to last section, I first analyze the entrepreneur's choice of equilibrium advertising as a function of the given quality level $\bar{S}$. Moving from fixed to endogenous quality helps me dual objectives. First, as in the base model, it helps to tease apart the direct effect of crowdfunding. Second, it helps to clearly delineate the complementary or mutually reinforcing nature of equilibrium advertising and quality decisions. The latter is critical in fully characterizing the nature of the indirect effect in this model.

With traditional entry, the entrepreneur first chooses the advertising level K_T and incurs the cost $\frac{{K_T}^3}{24}$. Then, she decides on price. Given the advertising level K_T, the entrepreneur's profit function is $\pi_T(\bar{S}, K_T, P_T) = K_T M\left(1 - \frac{P_T}{\bar{S}}\right) P_T - \frac{\bar{S}^2}{2} - \frac{{K_T}^3}{24}$. Similar to the previous scenario, the optimal price given quality is $P_T^* = \frac{\bar{S}}{2}$. Thus, the expected profit can be written as $E[\pi_T(\bar{S}, K_T)] = \frac{K_T \bar{S}}{8} - \frac{\bar{S}^2}{2} - \frac{{K_T}^3}{24}$.

If the entrepreneur launches her product through crowdfunding, she first chooses advertising level K_C by spending $\frac{{K_C}^3}{24}$ in stage 0. Then she sets the price P_C and the funding goal G to set up the crowdfunding campaign in stage 2. In stage 3, consumers choose whether to fund the entrepreneur at the posted price P_C. The entrepreneur's profit as a function of the advertising level K_C is

$$\pi_C(\bar{S}, K_C, P_C, G) = \begin{cases} K_C M\left(1 - \frac{P_C}{\bar{S}}\right) P - \frac{\bar{S}^2}{2} - \frac{{K_C}^3}{24}, K_C M\left(1 - \frac{P_C}{\bar{S}}\right) P_C \geq G \\ -\frac{{K_C}^3}{24}, K_C M\left(1 - \frac{P_C}{\bar{S}}\right) P_C < G \end{cases} \tag{7}$$

We obtain the optimal price and funding goal. Since M follows the uniform distribution over [0,1], the expected profit takes the following form $E[\pi_C(K_C)] = \frac{1}{8K_C}(K_C - 2\bar{S})^2 \bar{S} - \frac{{K_C}^3}{24}$.

Before, I go onto discuss optimal advertising strategy for a fixed quality I outline the direct effect of safeguarding in a crowdfunding platform. The direct effect of safeguarding accrues without endogenizing the advertising and quality decision. For a fixed advertising ($K_C = K_T = \overline{K}$) and quality one can see that $E[\pi_C(\overline{K})] > E[\pi_T(\overline{K})]$ and $E[\pi_C(\overline{K})] - E[\pi_T(\overline{K})] = 4s^3$, where the latter expression is the magnitude of direct effect. In other words, as one would expect the direct effect still persists in the full model.

Now coming back to the advertising decision for a fixed quality, the optimal level of advertising under traditional and crowdfunding market are given by:

$$K_T^*(\bar{S}) = \sqrt{\bar{S}} \tag{8}$$

$$K_C^*(\bar{S}) = \sqrt{\frac{1}{2}\bar{S}\left(1 + \sqrt{1 - 16\bar{S}}\right)} \tag{9}$$

The critical observation is that both $K_T(\bar{S})$ and $K_C(\bar{S})$ are increasing in $\bar{S}$. This is because higher quality enables the entrepreneur to charge a higher price and extract higher revenue from the informed consumers. Thus, with higher product quality, it is more valuable for the entrepreneur to spend on informing consumers. Second, also note that $K_T(\bar{S}) > K_C(\bar{S})$: for a given quality $\bar{S}$, the entrepreneur advertises less in the crowdfunding market than in the traditional market. Because of the all-or-nothing feature of crowdfunding, there is a chance for the entrepreneur to receive nothing (funding failure) in crowdfunding. That means the advertising spending will be wasted with a certain probability. The risk of funding failure deters the entrepreneur from spending more money on advertising.

Now consider the case when quality is endogenous: the entrepreneur makes the quality decision in the second stage after investing in advertising. Under traditional entry, in the second stage, the entrepreneur takes the advertising level K_T as given and sets quality S_T to maximize her expected profit $\frac{K_T S_T}{8} - \frac{{S_T}^2}{2} - \frac{{K_T}^3}{24}$. I obtain optimal decision variables as: is $K_T^* = \frac{1}{8}$, $S_C^* = \frac{1}{64}$, $P_C^* = \frac{1}{128}$, and $E[\pi_T^*] = \frac{1}{24576}$. In the crowdfunding case, the optimal decision variables are $K_C^* = \frac{4}{27}$, $S_C^* = \frac{2}{81}$, $P_C^* = \frac{1}{81}$, and $E[\pi_C^*] = \frac{16}{59049}$.

By simply comparing the optimal advertising levels in the two scenarios, I obtain Proposition 2.2, whose proof is contained in Appendix 17.

Proposition 2.2. The equilibrium advertising spending and quality are higher in the crowdfunding compared with that in the traditional market entry scenario. The expected profit is also higher through crowdfunding.

The equilibrium choice of advertising and quality is the source of indirect benefit from crowdfunding. First note that, $K_C^* > K_T^*$. In other words, with endogenous quality, the entrepreneur posts more ads when selling through a crowdfunded market relative to a traditional market. This contrasts with the result outlined in the model with exogenous quality where I noted $K_T(\bar{S}) > K_C(\bar{S})$ - the entrepreneur posted less informative ads when selling through a crowdfunded market than in a traditional market. Of course, equilibrium quality is also higher in line with the result in the previous section.

The intuition behind the result lies in the complementarity between the entrepreneur's quality and advertising strategies. In the model with endogenous quality, quality is set after advertising decision is being made. Therefore, one can compute the quality choice as a function of advertising which are $S_T^*(K_T) = \frac{K_T}{8}$ and $S_C^*(K_C) = \frac{K_C}{6}$. Notice both are increasing functions of advertising decisions, K_T and K_C. That is higher levels of advertising allows the entrepreneur to increase quality in equilibrium. Advertising is informative and hence is demand generating. Therefore, increasing advertising intensity also bolsters the entrepreneur's incentive to increase quality because a higher quality product (sold at a higher price) helps the entrepreneur extract more revenue from a larger addressable market. However, choice of higher quality, in turn makes the entrepreneur choose higher levels of informative ads in equilibrium. That is because, with higher product quality, it is more valuable for the entrepreneur to spend on informing consumers. As I saw before, both $K_T(\bar{S})$ and $K_C(\bar{S})$ are increasing in $\bar{S}$. Technically, if the entrepreneur knows that she will produce a higher quality product at Stage 2 of the game, it motivates her to advertise more in the Stage 0 so as to raise demand for the high quality product. Put differently, the complementarity goes as follows: Higher advertising increases the incentive to produce a higher quality product because a higher quality product allows greater *revenue extraction*. A higher quality product, in turn, raises the incentive to increase informative advertising in equilibrium because more informative advertising *expands demand*. This complementarity in choosing equilibrium advertising and quality increases profit. Now due to demand safeguards both advertising and quality choice and higher in the crowdfunding platform relative to traditional selling. Higher quality and advertising is at the heart of the indirect benefit from crowdfunding.

In summary, the core benefit from the demand safeguard with crowdfunding comprises of the direct effect and the indirect effect. The direct accrues because in crowdfunding the entrepreneur do not operate if the market condition is demonstrably poor. The indirect effect accrues through entrepreneur's choice of marketing mix elements (advertising, quality and price). It is affected by the aforementioned direct effect and also the complementarity between advertising and quality choice. Taken together, demand safeguards in crowdfunding platform can deliver offers higher profit to the entrepreneur than in the traditional channel.

Consumer (Funder) Surplus

Our analysis so far has shown that crowdfunding encourages entrepreneurs to produce more innovative products by providing a safeguard against uncertainty. For crowdfunding platforms such as Kickstarter or Indiegogo, the success of their business lies in serving both entrepreneurs and consumers where the consumers are also the funders. As Indiegogo once stated, "Indiegogo regularly develops and tests new features to meet the needs of both funders and campaign owners."[14] Therefore, it is imperative to consider not only the entrepreneur's profit but also the funders backing the entrepreneur. In this Section, I discuss results concerning consumer or the funder surplus.

In the traditional entry scenario, consumer surplus is

$$E[CS_T(K_T, S_T, P_T)] = \int_0^1 \int_{\frac{P_T}{S_T}}^1 K_T M(\theta S_T - P_T) d\theta \, dM \tag{10}$$

[14] Jon Russell, 2014, "Indiegogo Is Testing Optional Insurance Fees for Crowdfunded Products", https://techcrunch.com/2014/12/01/indiegogo-is-testing-optional-insurance-fees-for-crowdfunded-products/?ncid=rss. Accessed on 05/18/2018.

In the crowdfunding scenario, however, the consumer surplus is zero if the campaign fails to meet the funding goal. Only if the campaign is successfully funded, the consumer surplus takes the positive value $KM\int_{\frac{P}{S}}^{1}(\theta S-P)d\theta$. Let $\widetilde{M}=\frac{S_C G}{K_C P_C(S_C-P_C)}$ be the smallest market size that enables the campaign to be succeed. The consumer surplus is defined as follows.

$$CS_C(K_C,S_C,P_C,G,M)=\begin{cases} 0, & M<\widetilde{M} \\ K_C M\int_{\frac{P_C}{S_C}}^{1}(\theta S_C-P_C)d\theta\,, & M\geq\widetilde{M} \end{cases} \tag{11}$$

The ex-ante expected consumer surplus takes the following form.

$$E[CS_C(K_C,S_C,P_C,G)]=\int_{\widetilde{M}}^{1}\int_{\frac{P_C}{S_C}}^{1}K_C M(\theta S_C-P_C)d\theta\, dM \tag{12}$$

Given the above definitions, I compare the consumer surplus across the two market entry strategies. By substituting the entrepreneur's optimal quality, price, and advertising levels into the equation (10) and (12), I obtain Proposition 2.3.

Proposition 2.3. The expected consumer surplus is higher in crowdfunding.

The proof is presented in Appendix 18. The intuition behind the Proposition is driven by the fact that the aggregate consumer surplus comes from consumers who have a higher preference for quality (higher θ) - only consumers with higher ($\theta>\frac{P}{S}$) buy the product. That also implies that higher the equilibrium quality, higher is the consumer surplus. Indeed, in the previous section I noted that in equilibrium quality is higher in a crowdfunding platform than in a traditional market which explains the higher consumer surplus. It is interesting to notice that in the crowdfunded market, the entrepreneur enters only when $M\geq\frac{1}{3}$ (which can be obtained by substituting the equilibrium S_C,K_C and P_C in $\widetilde{M}$) and the entrepreneur do not enter the market when $M<\frac{1}{3}$. For $M<\frac{1}{3}$, the entrepreneur does operate in a traditional market. That implies for $M<\frac{1}{3}$, consumer surplus is higher with a traditional market (with surplus in crowdfunding being equal to zero). However, given the equilibrium choices of marketing mix elements, probability of the entrepreneur not entering (given by the probability of $M < \frac{1}{3}$ which is 1/3) is smaller than the probability of the firm entering (given by the probability of $M \geq \frac{1}{3}$ which is 2/3). With higher probability of entry and a higher a quality product, the consumers expect to derive higher surplus. In other words, while in general it is true that choosing a marginally higher quality marginally reduces the probability of market entry by increasing the funding goal, overall, equilibrium choices of advertising and quality are such that the probability of entering the market is higher than the probability of not entering the market. Factoring that in, the funders expected surplus is higher under crowdfunding than in traditional market.

6. Model Extensions

The model analysis has primarily focused on the benefits offered by crowdfunded market due to its AON feature and how such benefits translates to equilibrium strategies. In the following I undertake two extensions of the model. The basic thrust of the extensions is to highlight that there may be benefits to sell a product through the traditional channel over selling through crowdfunding. However, even after taking those into consideration, the core results qualitatively may persist to hold. Specifically, in the first extension, I relax the assumption that market uncertainty which is captured by the distribution of M being drawn from a uniform distribution $[0,1]$. Instead I introduce a parameter that allows me to vary the level of uncertainty associated with M. In a second extension, I relax the assumption that willingness to pay (captured by θ) for a crowdfunded product and a traditional product are the same.

6.1. Market Uncertainty

Uncertainty in the paper is captured by the assumption that the market size M is a draw from a uniform distribution $[0,1]$. In this section, I relax the assumption. Specifically, I assume that the market size M follows the uniform distribution over $\left[\frac{1}{2} - \beta, \frac{1}{2} + \beta\right]$. When β is high, the distribution of M has a larger spread that leads to a higher degree of uncertainty. When β is low, the degree of uncertainty is lower. Furthermore, I restrict the range of β to $\left[\frac{1}{4},\frac{1}{2}\right]$ to guarantee an interior solution.[15] In the following analysis, I examine the impact of β on quality, price, advertising, and profit.

In the traditional entry scenario, the entrepreneur makes marketing strategies based on the expected market size M. Since β does not influence the expectation of M, the entrepreneur's quality and price decision, as well as the profit expectation, remain the same as in the base model. However, the alternate assumptions affect the equilibrium in the crowdfunded market. Similar to previous sections, I can obtain the entrepreneur's expected profit.

$$E[\pi_C(K_C, S_C, G)] = \frac{1}{32\beta}\left(\frac{1}{2} + \beta - \frac{4G}{K_C S_C}\right) * \left(8G + K_C S_C + 2\beta K_C S_C - 4{S_C}^2\right) - \frac{{K_C}^3}{24} \tag{13}$$

It can be shown that the entrepreneur's optimal funding goal, quality, and advertising level are respectively $S_C^* = \frac{(1+2\beta)^4}{1296\beta}$, and $K_C^* = \frac{(1+2\beta)^3}{108\beta}$. The entrepreneur earns $E[\pi_C^*] = \frac{(1+2\beta)^9}{60466176\beta^3}$. In the following Proposition 2.4, I compare the profit in the crowdfunding and traditional markets. However, my primary focus is to outline and explain the effect of marginal changes in the uncertainty parameter, β of profit.

Proposition 2.4. In the traditional marketplace, the degree of uncertainty does not influence the entrepreneur's marketing strategy nor her profit expectation. In crowdfunding, the product quality, advertising level, as well as the entrepreneur's profit expectation increases in the degree of uncertainty. For any $\beta \in \left[\frac{1}{4},\frac{1}{2}\right]$, crowdfunding generates higher expected profit for the entrepreneur.

The last part of the proposition confirms the result that crowdfunding generates higher profit. One can again trace dichotomizing the excess profit from crowdfunding into direct and indirect benefits. The additional insights from this section comes from the first part of the proposition which outlines the effect of uncertainty. As mentioned previously, the equilibrium in the traditional market is invariant to the alternative assumption. On the contrary, the degree of uncertainty plays a significant role in crowdfunding, due to the safeguarding effect and quality/advertising enhancement effect. First, I find

[15] Relaxing this restriction does not change my results qualitatively.

that higher degree of uncertainty (β) enlarges the safeguarding effect, in effect suggesting that crowdfunding platform yields higher profit with marginal increase in uncertainty. This is because, with larger β, the entrepreneur is likely to either face a very large market or a very small market. In other words, both the downside risk and upside benefit are increasing in β which makes the safeguarding effect even more valuable.

To quantify the increased value of safeguarding more clearly, I compute the expected market size, conditional on the project being funded. For any given quality level S, advertising level K, and the corresponding optimal funding goal $G = \frac{S^2}{2}$, the entrepreneur only enters the market if the money raised through crowdfunding covers the funding goal (i.e., $KM\frac{S}{2} \geq G = \frac{S^2}{2}$). By rearranging the terms, I can rewrite the condition as $M \geq \frac{2S}{K}$. That means the funding campaign is only successful if the market size is no less than $\frac{2S}{K}$. Since M follows the uniform distribution over $\left[\frac{1}{2} - \beta, \frac{1}{2} + \beta\right]$, the conditional expectation of M given funding success is $E\left[M \middle| M \geq \frac{2S}{K}\right] = \frac{1}{4} + \frac{\beta}{2} + \frac{S}{K}$, which is increasing in β. Therefore, an increase in β provides a first order effect on expected market size and eventually on profit.

There is also a second order effect that gets channeled through equilibrium choices of advertising and quality. As mentioned above, with larger β the entrepreneur expects to face a larger demand conditional on the project being successfully funded. With a higher expected market size, the entrepreneur is more willing to advertise more and is also willing to produce a higher quality product in equilibrium. Technically, this can be seen from the fact that both $S^*(\beta) = \frac{1}{1296\beta}(1 + 2\beta)^4$ and $K^*(\beta) = \frac{1}{108\beta}(1 + 2\beta)^3$ are increasing functions of β. Now, higher $S^*(\beta)$ and $K^*(\beta)$ increases the expected market size given by $\frac{1}{4} + \frac{\beta}{2} + \frac{S}{K}$ even higher because $\frac{S^*(\beta)}{K^*(\beta)}$ is also increasing in β. In other words, the first order effect increases expected market size which leads to higher advertising and higher product quality. Higher advertising and higher quality increases the expected market size even further. Finally, a larger addressable market and a higher quality product yields a larger profit and the entrepreneur benefits from marginal increase in β.

To summarize, higher uncertainty comes with greater downside risk as well as more upside opportunity. Downside risk makes crowdfunding's safeguarding function more valuable, while upside opportunity strengthens the quality/advertising enhancement effect. In total, the two-part effect of a marginal increase in β, enables the entreprenuer to extract more revenue from the large demand, without worrying much about the market being very small. As a result, the expected profit increases when the market uncertainty is higher.

Finally, looking at the welfare implication of β. my result suggests that the expected consumer surplus $E[CS_C] = \frac{(1+2\beta)^9}{6718464\beta^3}$ in crowdfunding is increasing in β. Such an effect is driven by the high product quality associated with high β.

The above formulation can also be used to discern the effect of difference in potential market size in traditional and crowdfunded platforms. To wit, one may think that the market size in crowdfunding is smaller compared with traditional market. One possible reason is that the reach of crowdfunding platforms is not comparable to those of retail giants, such as Amazon. To capture that one can slightly alter the above formulation by assuming that M is a draw from uniform [0,1] when selling through traditional retail channel but M is drawn from uniform $\left[\frac{1}{2} - \beta, \frac{1}{2} + \beta\right]$ when selling through a crowdfunding platform. Moreover, to capture a smaller market potential with crowdfunding one can assume $\beta < 1/2$. One can again show that with a smaller market potential, the entrepreneur may be able to advertise more, produce a higher quality product and expect to make higher profits.[16]

6.2. Heterogeneity in Quality Preference

One difference between crowdfunding and traditional market consumers may lie in their preferences for the products. On the one hand, consumers in the crowdfunding scenario may have a lower valuation of the crowdfunded product because it is not immediately accessible. On the other hand, they may be willing to pay more in the crowdfunding context out of the noneconomic motivations (Mollick 2014). Such motivations include the community benefits crowdfunders get through interaction

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[16] The analysis is available from the author.

with the project owners (Belleflamme et al. 2013) or the desire to support entrepreneurship (i.e., the warm-glow effect in Hu et al. 2015). To account for the potential difference between the two populations, I assume that the quality preference θ follows different distributions in the traditional vs. crowdfunding scenario. In the traditional scenario, θ follows the same distribution as in the main model, the uniform distribution over $[0,1]$. In crowdfunding, I assume that θ is distributed uniformly over $[0, a]$. The value of a captures the difference in quality preference across the two scenarios. If $a < 1$, consumers, on average, value a project in crowdfunding less than that in the traditional market. On the contrary, consumers on average value the crowdfunded product more than that in the traditional market if $a > 1$. By incorporating the additional assumption of quality preference into the model, I can solve for the entrepreneur's optimal marketing strategies. The optimal product quality, price, and advertising level are respectively $S_C^* = \frac{2a^3}{81}$, $P_C^* = \frac{a^4}{81}$, and $K_C^* = \frac{4a^2}{27}$. As expected, all the strategic variables, price, product quality and advertising level are increasing in the parameter a. Comparing these values to those in the traditional scenario, I obtain Proposition 2.5.

Proposition 2.5. When consumers' quality preference θ is distributed uniformly over $[0, a]$ in crowdfunding, the product quality is higher in crowdfunding than in the traditional scenario if $a \geq \frac{3}{4}\left(\frac{3}{2}\right)^{1/3} \approx 0.859$. The price is higher in crowdfunding if $a \geq \frac{3}{2}\left(\frac{1}{8}\right)^{1/4} \approx 0.892$, and the advertising level is higher in crowdfunding if $a \geq \frac{3}{4}\left(\frac{3}{2}\right)^{1/2} \approx 0.919$. The expected profit is higher in crowdfunding if $a \geq \frac{3}{4}\left(\frac{3}{2}\right)^{1/2} \approx 0.919$. The consumer surplus is higher in crowdfunding if $a \geq \frac{3^{9/7}}{2^{15/7}} \approx 0.930$.

We see that the crowdfunding still enables the entrepreneur to produce higher quality product and conduct more advertising as long as the consumers' quality preference in the crowdfunding context is not too low compared with that in the traditional market. In other words, given some boundary market conditions, the core results of the paper hold with this alternate formulation.

The noteworthy result, however, relates to values of a that is sufficiently high but still is less than 1. Consider the region $0.919 \leq a < 1$. For $0.919 \leq a < 1$, even though the average willingness to pay for a product sold through crowdfunding platform is lower than the average willingness to pay for a product sold through the traditional market, the entrepreneur still makes a higher profit selling through the crowdfunding platform. For such high values, the entrepreneur is able to charge higher prices, advertise more and produce a higher quality product. The choice of higher strategic variables is even more interesting when gleaned from another angle. In both the crowdfunded and the traditional market, consumers with high quality preference ($\theta > P/S$) end up buying the product in equilibrium. However, maximum θ is higher in the traditional market than in the crowdfunded market because $0.919 \leq a < 1$. Put differently, even though in equilibrium demand is derived from the high end of the market and the high end is higher in the traditional market, crowdfunding nonetheless delivers higher profit. The reason why the entrepreneur is able to make a higher profit again rests on the safeguarding effect that increases the equilibrium choice of ads, quality, and price higher. Higher product quality drives up consumers' willingness to pay, while more advertising keeps more consumers informed. When $0.919 \leq a < 1$, the joint force of higher K and S overrides the negative impact of lower consumer preference. I can see this effect by focusing on the indifferent consumers in the traditional vs. crowdfunding scenarios. In the equilibrium, the indifferent consumer in the traditional market has a quality preference of $\overline{\theta_T} = \frac{1}{2}$, while the indifferent consumers' quality preference in crowdfunding is $\overline{\theta_C} = \frac{a}{2} < \overline{\theta_T}$. However, given the high product quality, the indifferent consumer in crowdfunding actually has higher willingness-to-pay for $0.919 \leq a < 1$. This can be seen from the fact that $\overline{\theta_C} S_C^* = \frac{a^4}{81} > \overline{\theta_T} S_T^* = \frac{1}{128}$. In other words, the indifferent customer is willing to pay more in the crowdfunding platform. Indeed for the same reason, willingness to pay is higher for the entire mass of potential demand - $[\overline{\theta_C}, a]$ in crowdfunding and $[\overline{\theta_T}, 1]$ in traditional. Moreover, higher K in crowdfunding ensures that, in equilibrium, a greater proportion of consumers with higher willingness to pay end up buying the product in the crowdfunded market vis-à-vis traditional market. Thus, even though $a < 1$, the entrepreneur is able to appropriate higher profit in the crowdfunded market. In summary, the core thrust of the result alludes to the overwhelming effect of safeguarding even when, on average, consumers are willing to pay less for a product from a crowdfunded market than in a traditional market.

Coming to consumer (or funder) surplus, once again, crowdfunding generates higher expected consumer surplus as long as a is greater than 0.930. It is worth noting that the lower value of a in crowdfunding, by definition, will drag down consumer surplus, since consumers now are harder to satisfy. Even so, the higher quality and more advertising in crowdfunding are able to boost the overall welfare of such consumers.

In summary, my results suggest the safeguarding effect, which allows the entrepreneur against uncertain market conditions in turn boosting the key strategic variables, persists under the alternate model formulation. In the following I discuss my key findings, outline some managerial implications and end with a brief note on limitations and some viable path for future research.

7. Summary and Conclusion

In their paper "Democratizing Innovation and Capital Access", Mollick and Robb (2016, pp.75) attest to one of the primary benefits of crowdfunding platforms - "Crowdfunding serves as an excellent tool for demonstrating demand, since it shows the willingness to pay for a product". Demonstration of demand de facto provides a safeguard against poor market conditions. Such a safeguard does not accrue if one sells a product through traditional channels. In traditional markets, even though one can use market research to gauge market demand, actual reactions from real buyers are not observed until the product is produced and sold. In this paper, I provide a simple micro foundation of demand safeguards provided by crowdfunding that allows me to tease out the effect of such safeguards on marketing mix elements - price, quality and advertising.

The core benefit from crowdfunding afforded by demand safeguards can be broken into two parts. The first effect is straightforward - crowdfunding delivers higher ex-ante profit by simply protecting the entrepreneur against poor market conditions. I call this the direct effect. The second effect is a little more nuanced. The protection against poor market allows the entrepreneur to advertise more and produce a higher quality product relative to a traditional channel. A higher quality product also allows the entrepreneur to charge a higher price, in turn, delivering higher profit. I call this the indirect effect of crowdfunding. Critical to the mechanics of the indirect effect is the complementarity of advertising and quality choices - to sell the higher quality product, the entrepreneur increases its advertising in equilibrium to raise awareness of its higher quality product. Moroever higher levels of advertising increases the incentive to make a higher quality product because higher quality enables greater revenue extraction. The source of the complementarity again harkens back to the demand safeguard provided in the crowdfunding platform. my results also suggest that consumer (funder) surplus are higher in a crowdfunded platform than in a traditional market. There are more funders that are informed in a crowdfunded platform and they receive a higher quality product leading to higher aggregate consumer surplus. The result that crowdfunding platform allows an entrepreneur to sell a relatively high quality product is consistent with Mollick and Nanda (2016). They find that, on average, the projects that are funded on kickstarter.com are of higher quality than the ones that are not funded.

The above result can be useful for innovators when unpacked differently. my results suggest than an entrepreneur with a high price - high quality product than what is currently available in the market may be better off selling the product through a crowdfunding platform rather than a traditional platform. This may especially be relevant for innovators with limited access to capital for whom downside risk of uncertain market conditions can be dooming. This interpretation comports well with some anecdotes. Take for instance the craft ice cream company MilkMaid Ice Cream started by Diana Hardeman. The company sells super premium all-natural ice cream. One can get a monthly subscription of $35 for just two pints of ice cream. One can also buy six pints for around $70. In a case study Glinska and Murray (2015) document the successful launch of MilkMaid through kickstarter.com. One rationale for the choice of a crowdfunding platform is, launching a premium product through a traditional channel requires raising funds from other sources such as from angel investors. Investors may perceive greater uncertainty supporting a venture from an innovator who is selling a high end product that comes with high premium. Crowdfunding is a particularly useful channel choice for such products. Demand safeguards delivers higher profit for entrepreneurs and higher surplus for funders. my results also suggest that the entrepreneur will be best served by launching an awareness advertising campaign. The campaign may help the entrepreneur to reap larger profit from a bigger addressable market.

The above issue of crowdfunding being profitable in highly uncertain markets is confirmed with an extension in which I solely focus on how marginal changes in uncertainty affects entrepreneur's marketing mix. my results suggest that in crowdfunded markets, increase in uncertainty increases profits for entrepreneurs and it also increases surplus for funders. This lends support to the result above that crowdfunding is a viable alternative in highly volatile markets. Finally, in another extension I show that even though average willingness to pay for consumers in the crowdfunding platform is lower relative to a traditional market, the entrepreneur may still be able to advertise more, sell a higher quality product, charge a higher a higher price and make higher ex-ante profit. In other words, the effects of demand safeguard that works thorough the marketing mix elements holds robust under various market modifications - even when the decks are stacked against operating in a crowdfunding platform.

Our model focuses on the particular form of reward-based crowdfunding to illustrate how an entrepreneur should take marketing strategies different from those in the traditional entry scenario. The entrepreneurs in my model can credibly commit to a product quality level in the crowdfunding stage. However, it is not always possible for entrepreneurs to make such credible commitments. The lack of credible commitment may give rise to the moral hazard problem: some entrepreneurs, after crowdfunding success, have the tendency to embezzle funds instead of investing properly in production (Strausz 2017). Thus, it will be interesting to extend the work to the situation where credible commitment is not possible and moral hazard issues exists. The role of crowdfunding as a two-sided platform is another issue that is worth exploring. In this paper, I focus on the interaction between the entrepreneur and the crowd. Under my conceptualization, the crowdfunding platform is just another channel to transfer goods from the entrepreneur to the crowd. In effect, this paper refrains from considering nuances of two-sided economics. It will therefore be interesting to analyze the issue of demand safeguards and marketing mix with a two-sided formulation.

Tables

Table 1. Characterization of Buyers in an Illustrative Example

Buyer segment	Size of buyer segment	Willingness-to-pay for a ticket	
		A plays in the game	*B* plays in the game
Segment 1	1	\$10	\$2
Segment 2	1	\$2	\$10

Table 2. Solutions without Capacity Constraint

Table 2a: Traditional Selling		
Parameter values	$\beta < \frac{1}{2}$	$\beta \geq \frac{1}{2}$
Optimal price	$\frac{1}{2}$	β
Sales	$\frac{1}{1-\beta}$	2
Revenue	$\frac{1}{2(1-\beta)}$	2β

Table 2b: Contingency Selling		
Parameter values	$\beta < \frac{1}{3}$	$\beta \geq \frac{1}{3}$
Optimal price for *A* ticket	$\frac{1}{2}$	$\frac{\beta}{1+\beta}$
Optimal price for *B* ticket	$\frac{1}{2}$	$\frac{\beta}{1+\beta}$
Sales for *A* ticket	$\frac{1}{2}$	1
Sales for *B* ticket	$\frac{1}{2}$	1
Revenue	$\frac{1}{2}$	$\frac{2\beta}{1+\beta}$

Table 3. Traditional Selling with Capacity Constraint

Parameter values	$\beta < \frac{1}{2}$		$\beta \geq \frac{1}{2}$
	$SC < \frac{1}{1-\beta}$	$SC \geq \frac{1}{1-\beta}$	
Optimal price	$1 - \frac{SC(1-\beta)}{2}$	$\frac{1}{2}$	$1 - \frac{SC(1-\beta)}{2}$
Sales	SC	$\frac{1}{1-\beta}$	SC
Revenue	$\frac{SC\left(2 - SC(1-\beta)\right)}{2}$	$\frac{1}{2(1-\beta)}$	$\frac{SC\left(2 - SC(1-\beta)\right)}{2}$

Table 4. Contingency Selling with Capacity Constraint

Parameter values	$\beta < \frac{1}{3}$		$\frac{1}{3} \le \beta < \frac{1}{2}$				$\beta \ge \frac{1}{2}$		
	$SC < \frac{1}{2}$	$SC \ge \frac{1}{2}$	$SC < \frac{1}{2}$	$\frac{1}{2} \le SC < 1 - \frac{\sqrt{3\beta - 1}}{2\sqrt{\beta}}$	$1 - \frac{\sqrt{3\beta - 1}}{2\sqrt{\beta}} \le SC < 1$	$1 \le SC$	$SC < 1 - \beta$	$1 - \beta \le SC < 1$	$1 \le SC$
optimal price for A ticket	$1 - SC$	$\frac{1}{2}$	$1 - SC$	$\frac{1}{2}$	$\frac{\beta(2 - SC)}{1 + \beta}$	$\frac{\beta}{1 + \beta}$	$1 - SC$	$\frac{\beta(2 - SC)}{1 + \beta}$	$\frac{\beta}{1 + \beta}$
optimal price for B ticket	$1 - SC$	$\frac{1}{2}$	$1 - SC$	$\frac{1}{2}$	$\frac{\beta(2 - SC)}{1 + \beta}$	$\frac{\beta}{1 + \beta}$	$1 - SC$	$\frac{\beta(2 - SC)}{1 + \beta}$	$\frac{\beta}{1 + \beta}$
sales for A ticket	SC	$\frac{1}{2}$	SC	$\frac{1}{2}$	SC	1	SC	SC	1
sales for B ticket	SC	$\frac{1}{2}$	SC	$\frac{1}{2}$	SC	1	SC	SC	1
revenue	$2SC(1 - SC)$	$\frac{1}{2}$	$2SC(1 - SC)$	$\frac{1}{2}$	$\frac{2\beta SC(2 - SC)}{1 + \beta}$	$\frac{2\beta}{1 + \beta}$	$2SC(1 - SC)$	$\frac{2\beta SC(2 - SC)}{1 + \beta}$	$\frac{2\beta}{1 + \beta}$

Table 5. Traditional Selling with Secondary Market and Neutral Price

Parameter values	$\beta < \frac{1}{2}$		$\beta \geq \frac{1}{2}$
	$SC < \frac{1}{1-\beta}$	$SC \geq \frac{1}{1-\beta}$	
Optimal price	$\frac{2 - SC + SC\beta}{2}$	$\frac{1}{2}$	$\frac{2 - SC + SC\beta}{2}$
Sales	SC	$\frac{1}{1-\beta}$	SC
Revenue	$\frac{SC(2 - SC + SC\beta)}{2}$	$\frac{1}{2(1-\beta)}$	$\frac{SC(2 - SC + SC\beta)}{2}$

Table 6. Contingency Selling with Secondary Market and Neutral Price

Parameter values	$SC < \frac{1-\beta}{2}$	$\frac{1-\beta}{2} \leq SC < \frac{3+\beta}{4}$	$SC \geq \frac{3+\beta}{4}$
optimal price for A ticket	$1 - SC$	$\frac{3 - 2SC + \beta}{4}$	$\frac{3+\beta}{8}$
optimal price for B ticket	$1 - SC$	$\frac{3 - 2SC + \beta}{4}$	$\frac{3+\beta}{8}$
sales for A ticket	SC	SC	$\frac{3+\beta}{4}$
sales for B ticket	SC	SC	$\frac{3+\beta}{4}$
Revenue	$2SC(1 - SC)$	$\frac{SC(3 - 2SC + \beta)}{2}$	$\frac{(3+\beta)^2}{16}$

Table 7. Traditional Selling with Secondary Market and Rubinstein Negotiation Price

Table 7a: $\gamma \geq \tilde{\gamma}$		
Parameter values	$SC < 1$	$SC \geq 1$
Resale price in state A or B	$\frac{1+(1-\gamma)\beta}{2-\gamma}$	$\frac{1+(1-\gamma)\beta}{2-\gamma}$
Secondary market trading volume in state A or B	$\frac{SC}{2}$	$\frac{1}{2}$
Firm's optimal price	$1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}$	$1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$
Sales	SC	1
Revenue	$\left(1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}\right)SC$	$1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$

Table 7b: $\gamma < \tilde{\gamma}$ & $\beta \geq \tilde{\beta}$			
Parameter values	$SC < 1$	$1 \leq SC < \widetilde{SC}$	$SC \geq \widetilde{SC}$
Resale price in state A or B	$\frac{1+(1-\gamma)\beta}{2-\gamma}$	$\frac{1+(1-\gamma)\beta}{2-\gamma}$	$\frac{1-\gamma+\beta}{2-\gamma}$
Secondary market trading volume in state A or B	$\frac{SC}{2}$	$\frac{1}{2}$	$\frac{SC}{2}$
Firm's optimal price	$1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}$	$1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$	$1-\frac{(1-\beta)\left((1-\gamma)SC+\gamma\right)}{2-\gamma}$
Sales	SC	1	SC
Revenue	$\left(1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}\right)SC$	$1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$	$\left(1-\frac{(1-\beta)\left((1-\gamma)SC+\gamma\right)}{2-\gamma}\right)SC$

Table 7. Traditional Selling with Secondary Market and Rubinstein Negotiation Price (Continued)

Table 7c: $\gamma < \tilde{\gamma}$ & $\beta < \tilde{\beta}$				
Parameter values	$SC < 1$	$1 \leq SC < \widetilde{SC}$	$\widetilde{SC} \leq SC < \frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}$	$SC \geq \frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}$
Resale price in state A or B	$\frac{1+(1-\gamma)\beta}{2-\gamma}$	$\frac{1+(1-\gamma)\beta}{2-\gamma}$	$\frac{1-\gamma+\beta}{2-\gamma}$	$\frac{1-\gamma+\beta}{2-\gamma}$
Secondary market trading volume in state A or B	$\frac{SC}{2}$	$\frac{1}{2}$	$\frac{SC}{2}$	$\frac{2-(2-\beta)\gamma}{(1-\beta)(1-\gamma)}$
Firm's optimal price	$1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}$	$1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$	$1-\frac{(1-\beta)\left((1-\gamma)SC+\gamma\right)}{2-\gamma}$	$\frac{2-(2-\beta)\gamma}{2(2-\gamma)}$
Sales	SC	1	SC	$\frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}$
Revenue	$\left(1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}\right)SC$	$1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$	$\left(1-\frac{(1-\beta)\left((1-\gamma)SC+\gamma\right)}{2-\gamma}\right)SC$	$\frac{(2-(2-\beta)\gamma)^2}{4(1-\beta)(1-\gamma)(2-\gamma)}$

$$*\tilde{\gamma} = \begin{cases} -\frac{2\left(-1+\beta+\sqrt{1-2\beta-\beta^2}\right)}{-3+3\beta+\sqrt{1-2\beta-\beta^2}}, & \beta < 0.4 \\ \frac{-1+3\beta}{\beta}, & \beta \geq 0.4 \end{cases}, \tilde{\beta} = \begin{cases} \frac{2(1-\gamma)}{4-3\gamma}, & \gamma < 0.5 \\ \frac{1}{3-\gamma}, & \gamma \geq 0.5 \end{cases}, \text{ and } \widetilde{SC} = \frac{2-(2-\beta)\gamma-\sqrt{\beta^2(4-8\gamma+5\gamma^2)+(8\beta-4)(1-\gamma)\gamma}}{2(1-\beta)(1-\gamma)}$$

Table 8. Contingency Selling with Decentralized Secondary Market

Parameter values	$SC < \frac{1-\beta-\gamma+\beta\gamma}{2-\gamma}$	$\frac{1-\beta-\gamma+\beta\gamma}{2-\gamma} \leq SC < \frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$	$SC \geq \frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$
optimal price for A ticket	$1-SC$	$\frac{3+\beta-\gamma-\beta\gamma}{4-2\gamma}-\frac{SC}{2}$	$\frac{3+\beta-\gamma-\beta\gamma}{4(2-\gamma)}$
optimal price for B ticket	$1-SC$	$\frac{3+\beta-\gamma-\beta\gamma}{4-2\gamma}-\frac{SC}{2}$	$\frac{3+\beta-\gamma-\beta\gamma}{4(2-\gamma)}$
sales for A ticket	SC	SC	$\frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$
sales for B ticket	SC	SC	$\frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$
Revenue	$2SC(1-SC)$	$SC\left(\frac{3+\beta-\gamma-\beta\gamma}{2-1\gamma}-SC\right)$	$\frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2}$

Table 9. Spot Selling

Parameter values	$\beta < \frac{1}{2}$		$\beta \geq \frac{1}{2}$		
	$SC < 1$	$SC \geq 1$	$SC < 1$	$1 \leq SC < \frac{1}{\beta}$	$\frac{1}{\beta} \leq SC \leq 2$
optimal price in state A	1	1	1	1	β
optimal price in state B	1	1	1	1	β
sales in state A	SC	1	SC	1	SC
sales in state B	SC	1	SC	1	SC
revenue	SC	1	SC	1	βSC

Appendices

Appendix 1. Derivation of the Results in Table 3 "Traditional Selling with Capacity Constraint"

Under traditional selling, the firm maximizes the following revenue function.

$$\pi_T(P) = \begin{cases} min\{2P, SC \cdot P\}, & P < \beta \\ min\left\{\frac{2P(1-P)}{1-\beta}, SC \cdot P\right\}, & \beta \le P < 1 \\ 0, & P \ge 1 \end{cases}$$

When $\beta \ge \frac{1}{2}$, the constraint is always binding. The firm's optimal price is $1 - \frac{SC(1-\beta)}{2}$, which results in a revenue of $\frac{SC(2-SC(1-\beta))}{2}$. When $\beta < \frac{1}{2}$, the constraint is only binding for $SC < \frac{1}{1-\beta}$. When $SC \ge \frac{1}{1-\beta}$, the firm charges the price $\frac{1}{2}$ and earns a revenue of $\frac{1}{2(1-\beta)}$.

Appendix 2. Derivation of the Results in Table 4 "Contingency Selling with Capacity Constraint"

Here, I take contingent ticket A as an example. The solution to the ticket B can be obtained in the same fashion. With capacity constraint SC, the revenue function for the A ticket takes the following form.

$$\pi_A(P_A) = \begin{cases} min\left\{\left(2 - \frac{1+\beta}{\beta}P_A\right)P_A, SC \cdot P_A\right\}, & P_A < \beta \\ min\{(1-P_A)P_A, SC \cdot P_A\}, & \beta \le P_A < 1 \\ 0, & P_A \ge 1 \end{cases}$$

When $\beta < \frac{1}{3}$, the SC constraint is binding for $SC < \frac{1}{2}$, the optimal price is $P_A^* = 1 - SC$. For $SC \ge \frac{1}{2}$, the constraint is not binding, and $P_A^* = \frac{1}{2}$.

When $\frac{1}{3} \leq \beta < \frac{1}{2}$ and $SC > \frac{1}{2}$, SC is binding, and $P_A^* = 1 - SC$. For $\frac{1}{2} \leq SC < 1 - \frac{\sqrt{3\beta-1}}{\sqrt{\beta}}$, SC is not binding and $P_A^* = \frac{1}{2}$. For $1 - \frac{\sqrt{3\beta-1}}{\sqrt{\beta}} \leq SC < 1$, SC is again binding and $P_A^* = \frac{\beta(2-SC)}{1+\beta}$. For $SC \geq 1$, the constraint is not binding and $P_A^* = \frac{\beta}{1+\beta}$.

When $\beta \geq \frac{1}{2}$, SC is binding for $SC < 1$. For $SC < 1 - \beta$, the firm sells only to segment 1 at $P_A^* = 1 - SC$. For $1 - \beta \leq SC < 1$, the firm sells to both segment 1 and 2 at $P_A^* = \frac{\beta(2-SC)}{1+\beta}$. For $SC \geq 1$, SC is not binding and $P_A^* = \frac{\beta}{1+\beta}$. Similarly, I obtain P_B^* for contingent B ticket and total revenue $\pi_C = \pi_A + \pi_B$.

Appendix 3. Proof of Proposition 1.2a

First, I look at the case $\beta < \frac{1}{3}$. For $SC < \frac{1}{2}$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = 2SC(1 - SC)$. For $\frac{1}{2} \leq SC < \frac{1}{1-\beta}$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = \frac{1}{2}$ holds if $SC < \overline{SC} = \frac{1-\sqrt{\beta}}{1-\beta}$. It can be verified that $\overline{SC}$ lies in the range $\left[\frac{1}{2}, \frac{1}{1-\beta}\right]$.

Second, I turn to the case $\frac{1}{3} \leq \beta < \bar{\beta} \approx 0.420$. When $SC < \frac{1}{2}$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = 2SC(1 - SC)$. When $\frac{1}{2} \leq SC < 1 - \sqrt{\frac{3\beta-1}{4\beta}}$, the contingency selling revenue is $\frac{1}{2}$, while the traditional selling revenue is $SC\left(1 - \frac{SC(1-\beta)}{2}\right)$. The contingency selling generates higher revenue if $SC < \overline{SC} = \frac{1-\sqrt{\beta}}{1-\beta}$. It can be verified that $\overline{SC} = \frac{1-\sqrt{\beta}}{1-\beta}$ lies in the range $\left[\frac{1}{2}, 1 - \sqrt{\frac{3\beta-1}{4\beta}}\right]$.

Third, I look at the case $\bar{\beta} \leq \beta < \frac{1}{2}$. When $SC < \frac{1}{2}$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = 2SC(1 - SC)$. When $\frac{1}{2} \leq SC < 1 - \sqrt{\frac{3\beta-1}{4\beta}}$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = \frac{1}{2}$. When $1 - \sqrt{\frac{3\beta-1}{4\beta}} \leq SC < 1$,

$\pi_T = SC\left(1-\frac{SC(1-\beta)}{2}\right) < \pi_C = \frac{2SC(2-SC)\beta}{1+\beta}$ if $SC < \overline{SC} = \frac{2(3\beta-1)}{-1+4\beta+\beta^2}$. It can be verified that $\overline{SC} = \frac{2(3\beta-1)}{-1+4\beta+\beta^2}$ lies in the range $\left[1-\sqrt{\frac{3\beta-1}{4\beta}}, 1\right]$.

Finally, I examine the case $\beta \geq \frac{1}{2}$. For $SC < 1-\beta$, $\pi_T = SC\left(1-\frac{SC(1-\beta)}{2}\right) < \pi_C = 2SC(1-SC)$. For $1-\beta \leq SC < 1$, $\pi_T = SC\left(1-\frac{SC(1-\beta)}{2}\right) < \pi_C = \frac{2SC(2-SC)\beta}{1+\beta}$ if $SC < \overline{SC} = \frac{2(3\beta-1)}{-1+4\beta+\beta^2}$. It can be verified that the threshold $\overline{SC} = \frac{2(3\beta-1)}{-1+4\beta+\beta^2}$ lies in the range $[1-\beta, 1]$.

Appendix 4. Proof of Proposition 1.2b

By differentiating $\overline{SC}$ with respect to β, I obtain the first order derivative of $\overline{SC}$.

$$\frac{\partial\overline{SC}}{\partial\beta} = \begin{cases} -\dfrac{1}{2\left(1-\sqrt{\beta}\right)^2\sqrt{\beta}}, & \beta < \bar{\beta} \\ \dfrac{2+4\beta-6\beta^2}{(-1+4\beta+\beta^2)^2}, & \beta \geq \bar{\beta} \end{cases}$$

It is easy to check that $\frac{\partial\overline{SC}}{\partial\beta}$ is negative for $\beta < \bar{\beta}$ and positive for $\beta \geq \bar{\beta}$.

Appendix 5. Derivation of the Results in Table 5 "Traditional Selling with Secondary Market and Neutral Price"

First, I derive the following demand function of the general ticket.

$$Q(P_T) = \begin{cases} 1+\dfrac{p-P_T}{p-\beta}, & P \leq p \\ \dfrac{1-P_T}{1-p}, & P > p \end{cases}$$

If I denote x to the demand in segment 1 and y to the demand in segment 2, it follows immediately that $x+y=Q$. It is obvious that in an equilibrium, either $Q<1$ or $Q \geq 1$ must hold.

We first look at the case $Q < 1$. For the indifferent consumer θ in segment 1, the net valuations from "*buy now*" and "*buy later*" must equal in the equilibrium.

$$\rho_\theta + (1 - \rho_\theta)p - P_T = \rho_\theta(1 - p)\frac{y}{1 - x}$$

The expression $\frac{y}{1-x}$ is the chance that consumer from segment 1 can get a ticket in the secondary market if the true state is A. Since ρ follows the uniform distribution on $[0, 1]$, I obtain $1 - \rho_\theta = x$. By substituting ρ_θ with $1 - x$ and rearranging terms, I have $x + y = \frac{1-P_T}{1-p}$. Since $x + y = Q$, I have $Q(P_T) = \frac{1-P_T}{1-p}$. For $Q < 1$ to hold, P_T must be greater than p.

Next, I turn to the case $Q \geq 1$. Similar to the previous case, I obtain the following equality for the indifferent consumer θ in segment 1.

$$\rho_\theta + \frac{1 - y}{x}(1 - \rho_\theta)p + \left(1 - \frac{1 - y}{x}\right)(1 - \rho_\theta)\beta - P_T = \rho_\theta(1 - p)$$

$\frac{1-y}{x}$ refers to the probability that a consumer with a ticket is able to sell her ticket in the secondary market if the true state is B. By replacing ρ_θ with $1 - x$ and rearranging terms, I get $Q(P_T) = 1 + \frac{p-P_T}{p-\beta}$. For $Q \geq 1$ to be true, P_T must be no greater than p. Thus, I have the demand function in the follow form.

$$Q(P_T) = \begin{cases} 1 + \dfrac{p - P_T}{p - \beta}, & P_T \leq p \\ \dfrac{1 - P_T}{1 - p}, & P_T > p \end{cases}$$

Since $p = \frac{1+\beta}{2}$, the above demand function can be expressed as $Q(P_T) = \frac{2(1-P_T)}{1-\beta}$. Then, I obtain the profit function, $\pi_T(P_T) = \frac{2(1-P_T)P_T}{1-\beta}$. The firm's optimal pricing strategy is to charge $P_T^* = \frac{2-SC+SC\beta}{2}$ if $SC \leq min\left\{\frac{1}{1-\beta}, 2\right\}$ and $P_T^* = \frac{1}{2}$ otherwise. The results in Table 5 have been derived.

Appendix 6. Derivation of the Results in Table 6 "Contingency Selling with Secondary Market and Neutral Price"

Throughout this proof, I will take A tickets as the example. The solution for B tickets can be obtained in the same fashion. I first discuss the scenario when the firm only sells the contingent tickets of each type to its corresponding segment. All the A tickets are sold to segment 1 consumers. Just as in the main model, the demand function would be $Q(P_A) = 1 - P_A$. This scenario holds as long as $P_A \geq p$. This is because when the primary price is lower than the resale price, segment 2 consumers cannot gain anything by buying and reselling A tickets. It can be verified that it is optimal for the firm to price at $P_A = 1 - SC$ and sells to only segment 1 as long as $SC < 1 - p = \frac{1-\beta}{2}$.

Second, I analyze the scenario where the firm sells the contingent tickets of each type to both consumer segments. Let me denote x to the demand in segment 1 and y to the demand in segment 2, it follows immediately that either $x + y < 1$ or $x + y \geq 1$. I discuss the case $x + y < 1$ first. For a consumer i in segment 1, her net valuation from buying A ticket now is $\rho_i - P_A$, while her net valuation from "*buy later*" is $\rho_i(1-p)\frac{y}{1-x}$. For a consumer j in segment 2, her net valuation for A ticket is $\rho_j p - P_A$, while her net valuation from "*buy later*" is 0.

Under any price P_A, the net utility from "buy now" and "buy later" must equal for the indifferent consumer θ in segment 1 and the indifferent consumer μ in segment 2.

$$\begin{cases} \rho_\theta - P_A = \rho_\theta(1-p)\dfrac{y}{1-x} \\ \rho_\mu p - P_A = 0 \end{cases}$$

In segment 1, all the consumers with $\rho \geq \rho_\theta$ will pruchase the ticket, while in segment 2, all the consumers with $\rho \geq \rho_\mu$ will purchase the ticket. Thus, I can replace ρ_θ with $1 - x$ and ρ_μ with $1 - y$.

$$\begin{cases} 1 - x - P_A = (1-p)y \\ (1-y)p - P_A = 0 \end{cases}$$

By adding up these two equations, I obtain $x + y = 1 + p - 2P_A$. Thus, the demand function becomes $Q(P_A) = 1 + p - 2P_A$. And the profit function is thus $\pi(P_A) = (1 + p - 2P_A)P_A$. Since $p = \frac{1+\beta}{2}$, the profit function can be rewritten as $\pi(P_A) = \left(1 + \frac{1+\beta}{2} - 2P_A\right)P_A$. It turns my that the optimal price is $P_A = \frac{3-2SC+\beta}{4}$ if $SC < \frac{3+\beta}{4}$ and $P_A = \frac{3+\beta}{8}$ otherwise. This holds if $\frac{1-\beta}{2} \leq SC < 1$. Within this range, the highest revenue the firm can gain is $\frac{(3+\beta)^2}{16}$.

Next, I prove that the revenue cannot be higher than $\frac{(3+\beta)^2}{16}$ if the firm sells $x + y \geq 1$ tickets. When $x + y \geq 1$, the net valuation from "buy now" and "buy later" is $\rho_i - P_A$ and $\rho_i(1-p)$ for a consumer i in segment 1. For a consumer j in segment 2, her net valuation for A ticket is $\rho_j \frac{1-x}{y} p + \rho_j \left(1 - \frac{1-x}{y}\right)\beta - P_A$, while her net valuation from "*buy later*" is 0. Under any price P, for the indifferent consumer, the net utility from "buy now" and "buy later" must equal for the indifferent consumer θ in segment 1 and the indifferent consumer μ in segment 2.

$$\begin{cases} \rho_\theta - P_A = \rho_\theta(1-p) \\ \rho_\mu \dfrac{1-x}{y} p + \rho_\mu \left(1 - \dfrac{1-x}{y}\right)\beta - P_A = 0 \end{cases}$$

Similar to the previous analysis, I can replace ρ_θ and ρ_μ with $1 - x$ and $1 - y$.

$$\begin{cases} 1 - x - P_A = (1-x)(1-p) \\ (1-y)\dfrac{1-x}{y} p + (1-y)\left(1 - \dfrac{1-x}{y}\right)\beta - P_A = 0 \end{cases}$$

By solving these two equations simultaneously, I obtain the following demand function.

$$Q(P_A) = \frac{3\beta + 3\beta^2 - 2P_A(1+2\beta) + \sqrt{4{P_A}^2 - 4P_A\beta^2(1+\beta) + \beta^2(1+\beta)^2}}{2\beta(1+\beta)}$$

Let me suppose the firm can earn a revenue higher than $\frac{(3+\beta)^2}{16}$ by selling $Q = x + y \geq 1$ tickets. Then, it must be the case that $P_A > \frac{(3+\beta)^2}{16(x+y)}$. When $P_A > \frac{(3+\beta)^2}{16(x+y)}$, the demand $Q(P_A)$ must be smaller than $\frac{-9-24\beta+24\beta(x+y)-13\beta^2+24\beta^2(x+y)-2\beta^3+8(x+y)\sqrt{\frac{(3+\beta)^4}{64(x+y)^2}-\frac{\beta^2(1+\beta)(3+\beta)^2}{4(x+y)}+\beta^2(1+\beta)^2}}{16\beta(1+\beta)(x+y)}$.

It can be shown that the above expression is never greater than $x + y$ for $x + y \in [1, 2]$. That means, to sell $x + y$ tickets, the firm cannot set the price higher than $\frac{(3+\beta)^2}{16(x+y)}$. As a result, the firm cannot earn a revenue higher than $\frac{(3+\beta)^2}{16}$ by selling $Q = x + y \geq 1$ tickets. In a word, the firm's optimal price is $P_A = \frac{3+\beta}{8}$ for $SC > 1$.

To summarize, the optimal price for ticket A is $P_A = 1 - SC$ for $SC < \frac{1-\beta}{2}$, $P_A = \frac{3-2SC+\beta}{4}$ for $\frac{1-\beta}{2} \leq SC < \frac{3+\beta}{4}$, and $P_A = \frac{3+\beta}{8}$ for $SC \geq \frac{3+\beta}{4}$. In the similar fashion, the optimal price for ticket B can be obtained.

Appendix 7. Proof of Proposition 1.3

First, I show that contingency selling always outperforms traditional selling for $SC < 1$. Specifically, for $SC < \frac{1-\beta}{2}$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = 2SC(1 - SC)$. For $\frac{1-\beta}{2} \leq SC < \frac{3+\beta}{4}$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = \frac{SC(3-2SC+\beta)}{2}$. For $\frac{3+\beta}{4} \leq SC < 1$, $\pi_T = SC\left(1 - \frac{SC(1-\beta)}{2}\right) < \pi_C = \frac{(3+\beta)^2}{16}$.

Second, I turn to the case $SC \geq 1$, where $\pi_C = \frac{(3+\beta)^2}{16}$. Under traditional selling, π_T is $\frac{SC(2-SC+SC\beta)}{2}$ for $SC < \frac{1}{1-\beta}$ and $\frac{1}{2(1-\beta)}$ otherwise. When SC is smaller (greater) than $\frac{4-\sqrt{2(-1+3\beta+5\beta^2+\beta^3)}}{4-4\beta}$, the contingency selling revenue $\frac{(3+\beta)^2}{16}$ is higher (lower) than $\frac{SC(2-SC+SC\beta)}{2}$. It can be verified that $\frac{4-\sqrt{2(-1+3\beta+5\beta^2+\beta^3)}}{4-4\beta}$ lies in the range $\left[1, \frac{1}{1-\beta}\right]$ as long as $\beta \geq \sqrt{5}-2$. If $\beta < \sqrt{5}-2$, the contingency selling revenue is always higher than the traditional selling revenue.

Appendix 8. Proof of Lemma 1 "Resale Price through Bargaining"

Lemma 1 is a modified version of the classic result in Osborne and Rubinstein (1990, Chapter 6). Formally, the equilibrium is defined as follows:

A combination of $p(S,B)$, $V_S(S,B)$, and $V_B(S,B)$ is an equilibrium if it satisfies the following conditions.

First, for $\varDelta \longrightarrow 0$,

$$\begin{cases} \big(1-p(S,B)\big) - \gamma V_B(\varDelta, B-S+\varDelta) = \big(p(S,B)-\beta\big) - \gamma V_S(\varDelta, B-S+\varDelta), |S < B \\ \big(1-p(S,B)\big) - \gamma V_B(\varDelta,\varDelta) = \big(p(S,B)-\beta\big) - \gamma V_S(\varDelta,\varDelta), |S = B \\ \big(1-p(S,B)\big) - \gamma V_B(S-B+\varDelta,\varDelta) = \big(p(S,B)-\beta\big) - \gamma V_S(S-B+\varDelta,\varDelta), |S > B \end{cases}$$

Second,

$$\begin{cases} V_S(S,B) = p(S,B) - \beta \text{ and } V_B(S,B) = \frac{S}{B}\big(1-p(S,B)\big), |S < B \\ V_S(S,B) = p(S,B) - \beta \text{ and } V_B(S,B) = 1 - p(S,B), |S = B \\ V_S(S,B) = \frac{B}{S}\big(p(S,B)-\beta\big) \text{ and } V_B(S,B) = 1 - p(S,B), |S > B \end{cases}$$

The first condition ensures that $p(S,B)$ is the Nash solution given the surplus that a buyer-seller pair expects to gain if it deviates from the equilibrium. Note that in a continuum of consumer distribution, each individual counts for only a negligible portion, $\varDelta \longrightarrow 0$. In the case $S < B$, there will be $\varDelta + B - S$ buyers and $\varDelta$ seller in the market if a pair of matched buyer and seller deviates from the equilibrium. Thus, $V_S(\varDelta, B-S+\varDelta)$ and $V_B(\varDelta, B-S+\varDelta)$ are the seller's and the buyer's respective expected surplus if they cannot reach an agreement. The second condition ensures that it is rational for any buyer and seller, once matched, to accept the negotiated price $p(S,B)$.

The proof for $S < B$ goes as follow. When there are more buyers than sellers in the secondary market, the equilibrium resale price must satisfy $(p(S,B) - \beta) - \gamma V_S(\Delta, B - S + \Delta) = \big(1 - p(S,B)\big) - \gamma V_B(\Delta, B - S + \Delta)$ according to the aforementioned definition of equilibrium. Respectively, $V_S(\Delta, B - S + \Delta)$ and $V_B(\Delta, B - S + \Delta)$ are the seller's and buyer's expected surpluses if they cannot reach an agreement in the first round and stay active in the market for the second round. They serve as the disagreement values in the first-round bargaining process. To solve for p, I first solve for $V_S(\Delta, B - S + \Delta)$ and $V_B(\Delta, B - S + \Delta)$. Thus, I need to examine the equilibrium condition in the second round. For any matched buyer and seller, if they cannot reach an agreement in the first round, they enter the second round. Given that all the other individuals behave according to the equilibrium, there will be $\Delta + B - S$ active buyers and Δ active sellers in the second round, with $\Delta \longrightarrow 0$. In the second round, the probability to enter a match is $\frac{\Delta}{\Delta + B - S}$ for the buyer and 1 for the seller. If the matched pair cannot reach an agreement in the second round, there will still be $\Delta + B - S$ buyers and Δ sellers remaining active in the market. Thus, their expected payoff of entering the third round are still $V_S(\Delta, B - S + \Delta)$ and $V_B(\Delta, B - S + \Delta)$. For $p(\Delta, B - S + \Delta)$ to be the equilibrium price in the second round, the following system of equations must be satisfied:

$$\begin{cases} (p(\Delta, B - S + \Delta) - \beta) - \gamma V_S(\Delta, B - S + \Delta) = \big(1 - p(\Delta, B - S + \Delta)\big) - \gamma V_B(\Delta, B - S + \Delta) \\ p(\Delta, B - S + \Delta) - \beta = V_S(\Delta, B - S + \Delta) \\ \dfrac{\Delta}{B - S + \Delta}\big(1 - p(\Delta, B - S + \Delta)\big) = V_B(\Delta, B - S + \Delta) \end{cases}$$

By solving the above system of equations, I obtain

$$\begin{cases} V_S(\Delta, B - S + \Delta) = \dfrac{1 - \beta}{2 - \gamma} \\ V_B(\Delta, B - S + \Delta) = 0 \end{cases}$$

By plugging these values into the equilibrium condition of the first round, I get $p(S,B) = \frac{1 + (1 - \gamma)\beta}{2 - \gamma}$ for $S < B$. Similarly one can derive the equilibrium resale market bargained prices for $S = B$ and $S > B$.

Appendix 9. Derivation of the Results in Table 7 "Traditional Selling with Secondary Market and Rubenstein Negotiation Price"

We derive the results in Table 7 in 3 steps. First, I solve for the case $SC < 1$. Following the same logic as 'Derivation of Results in Table 5" above, I can show that the demand function is $Q(P_T) = \frac{1-P_T}{1-p}$ for $SC < 1$. Since $SC < 1$ and total market size is 2, there will be more buyers than sellers regardless of which state eventually occurs. According to Lemma 1, the resale price is $p = \frac{1+(1-\gamma)\beta}{2-\gamma}$ no matter which state (A or B) occurs. Thus, the demand function can be rewritten as $Q(P_T) = \frac{(2-\gamma)(1-P_T)}{1-\gamma-(1-\gamma)\beta}$. It can be shown that $P_T^* = 1 - \frac{(1-\beta)(1-\gamma)SC}{2-\gamma}$ for $SC < 1$.

Second, I turn to the case $SC = 1$, where multiple equilibria exist. I look for the equilibrium that generates the highest revenue for the firm.[17] When $SC = 1$, the equilibrium resale price depends on the sales in the first period. If the firm sells to its full capacity 1, then the resale price will be $p = \frac{1+\beta}{2}$ since there will be equal amounts of buyers and sellers in the secondary market. Then, following the same procedure as in the derivation of Table 5, I obtain the demand function $Q(P_T) = \frac{2(1-P_T)}{1-\beta}$. To sell up to the full capacity, the firm has to charge $P_T = \frac{1+\beta}{2}$. The resulting revenue is $\pi_T = \frac{1+\beta}{2}$.

If instead the firm sells less than its full capacity 1, the resale price will be $\frac{1+(1-\gamma)\beta}{2-\gamma}$, since there will be fewer sellers than buyers in the secondary market. Then, the demand function is $Q(P_T) = \frac{(2-\gamma)(1-P_T)}{1-\gamma-(1-\gamma)\beta}$, which leads to $P_T^* = 1 - \frac{(1-\beta)(1-\gamma)SC}{2-\gamma}$. As long as $P_T > 1 - \frac{(1-\beta)(1-\gamma)}{2-\gamma}$, the demand will be smaller than capacity. Thus, the highest possible revenue occurs when P_T is close to but still greater than $1 - \frac{(1-\beta)(1-\gamma)}{2-\gamma}$. As a result, $\pi_T^* = \lim_{P \to 1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}} \frac{(2-\gamma)(1-P)}{1-\gamma-(1-\gamma)\beta} P = 1 - \frac{(1-\beta)(1-\gamma)}{2-\gamma}$.

Third, I turn to the case that $SC > 1$. When $SC > 1$, the firm faces two possible pricing schemes. First, the firm can charge a high price to serve a few consumers so that the resale price stays high as if

[17] my key qualitative conclusion still holds for other equilibria.

$SC \leq 1$. Second, the firm can charge a low price to sell more than one tickets. To solve for the second scheme, I obtain the demand function $Q(P) = 1 + \frac{p-P}{p-\beta}$ for $SC > 1$. For $SC > 1$, , there will be fewer buyers than sellers in the secondary market regardless of which state (A or B) occurs. According to Lemma 1, the resale price is $\frac{1-\gamma+\beta}{2-\gamma}$. Given this resale price, the demand function can be rewritten as $Q(P) = 2 - \frac{(2-\gamma)(P-\beta)}{(1-\beta)(1-\gamma)}$, with the profit function being $\pi(P) = \left(2 - \frac{(2-\gamma)(P-\beta)}{(1-\beta)(1-\gamma)}\right)P$. The optimal price turns to be $P = 1 - \frac{(1-\beta)((1-\gamma)SC+\gamma)}{2-\gamma}$ if $SC < \frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}$ and $P = \frac{2-(2-\beta)\gamma}{2(2-\gamma)}$ otherwise. That leads to a revenue of $\left(1 - \frac{(1-\beta)((1-\gamma)SC+\gamma)}{2-\gamma}\right)SC$ if $SC < \frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}$ and $\frac{(2-(2-\beta)\gamma)^2}{4(1-\beta)(1-\gamma)(2-\gamma)}$ otherwise.

Now, let me compare the revenue generated from the two schemes. The first scheme generates $1 - \frac{(1-\beta)(1-\gamma)}{2-\gamma}$, while the second scheme generates $min\left\{\left(1 - \frac{(1-\beta)((1-\gamma)SC+\gamma)}{2-\gamma}\right)SC, \frac{(2-(2-\beta)\gamma)^2}{4(1-\beta)(1-\gamma)(2-\gamma)}\right\}$. When $\gamma > \tilde{\gamma}$, then the first scheme generates higher revenue for any $SC \in [1,2]$. Within this range, the firm targets only the high valuation segment. When $\beta \geq \frac{1-2\gamma-3\rho+3\gamma\rho}{-1-\gamma-3\rho+3\gamma\rho}$, then the first (or second) scheme generates higher revenue for $SC < \frac{2-(2-\beta)\gamma-\sqrt{\beta^2(4-8\gamma+5\gamma^2)+(8\beta-4)(1-\gamma)\gamma}}{2(1-\beta)(1-\gamma)}$ (or $SC \geq \frac{2-(2-\beta)\gamma-\sqrt{\beta^2(4-8\gamma+5\gamma^2)+(8\beta-4)(1-\gamma)\gamma}}{2(1-\beta)(1-\gamma)}$). Thus, the firm chooses to sell less than one tickets if $SC < \frac{2-(2-\beta)\gamma-\sqrt{\beta^2(4-8\gamma+5\gamma^2)+(8\beta-4)(1-\gamma)\gamma}}{2(1-\beta)(1-\gamma)}$ and sells more than one tickets otherwise.

Appendix 10. Derivation of the Results in Table 8 "Contingency Selling with Decentralized Secondary Market"

Without loss of generality I take the example of A ticket. I give the solution to the contingency selling case in two steps. I first look at the case $SC < 1 - p_A$. When $SC < 1 - p_A$, one can use the logic akin to "Derivation of Results in Table 6" to show that all the A tickets go to segment 1 consumers, and no resale happens in the secondary market. Thus, the optimal price(s) are the same as those in the Table 6 ($P_A = 1 - SC$).

Second, I turn to the case $1 - p_A \leq SC < 1$. Within this range, the firm sells the contingent tickets of each type to both consumer segments, while knowing that the will be more buyers more sellers in the secondary market. As shown in "Derivation of Results in Table 6", the profit function is thus $\pi(P_A) = (1 + p_A - 2P_A)P_A$. Since $SC < 1$, I have $p_A = \frac{1+(1-\gamma)\beta}{2-\gamma}$ and rewrite the profit function as $\pi(P_A) = \left(1 + \frac{1+(1-\gamma)\beta}{2-\gamma} - 2P_A\right)P_A$. The optimal price is $P_A = \frac{3+\beta-\gamma-\beta\gamma}{4-2\gamma} - \frac{SC}{2}$ if $SC < \frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$ and $P_A = \frac{3+\beta-\gamma-\beta\gamma}{4(2-\gamma)}$ otherwise. Within the range $1 - p_A \leq SC < 1$, the highest π_A is $\frac{(3+\beta-\gamma-\beta\gamma)^2}{8(2-\gamma)^2}$.

Finally, I show the firm cannot earn more than $\frac{(3+\beta-\gamma-\beta\gamma)^2}{8(2-\gamma)^2}$ if it sells more than one A tickets. I know that when the firm sells more than one tickets, the profit function takes the following form.

$$\pi_A(P_A) = \left(\frac{3\beta p_A - P_A(\beta + 2p_A) + \sqrt{4P_A p_A(p_A - \beta)\beta + \left(p_A\beta + P_A(\beta - 2p_A)\right)^2}}{2\beta p_A}\right)P_A$$

$\pi_A(P_A)$ is increasing in p, for $p \in \left[0, \frac{1+\beta}{2}\right]$. It means for any P_A value, the highest revenue is achieved when $p = \frac{1+\beta}{2}$. Note that when $p = \frac{1+\beta}{2}$, the model is identical to the previous case with neutral resale price, where the maximal π_A is $\frac{(3+\beta)^2}{32}$. Since $\frac{(3+\beta)^2}{32} \leq \frac{(3+\beta-\gamma-\beta\gamma)^2}{8(2-\gamma)^2}$, I know the firm cannot earn more than $\frac{(3+\beta-\gamma-\beta\gamma)^2}{8(2-\gamma)^2}$ by selling more than one A tickets.

To summarize, the optimal price for ticket A is $P_A = 1 - SC$ for $SC < \frac{1-\beta-\gamma+\beta\gamma}{2-\gamma}$, $P_A = \frac{3+\beta-\gamma-\beta\gamma}{4-2\gamma} - \frac{SC}{2}$ for $\frac{1-\beta-\gamma+\beta\gamma}{2-\gamma} \leq SC < \frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$, and $P_A = \frac{3+\beta-\gamma-\beta\gamma}{4(2-\gamma)}$ for $SC \geq \frac{3+\beta-\gamma-\beta\gamma}{2(2-\gamma)}$. In the similar fashion, the optimal price for ticket B can be obtained.

Appendix 11. Proof of Proposition 1.4

First, I show that contingency selling always outperforms traditional selling for $SC < 1$. Specifically, when $SC < \frac{1-\beta-\gamma+\beta\gamma}{2-\gamma}$, the contingency selling revenue is $2SC(1 - SC)$, while the traditional selling revenue is $\left(1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}\right)SC$. The former is higher than the latter for $SC < \frac{1-\beta-\gamma+\beta\gamma}{2-\gamma}$. When $\frac{1-\beta-\gamma+\beta\gamma}{2-\gamma} \leq SC < \frac{3+\beta-2\gamma}{4-2\gamma}$, the contingency selling revenue is $SC\left(\frac{3+\beta-\gamma-\beta\gamma}{2-1\gamma}-SC\right)$, while the traditional selling revenue is $\left(1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}\right)SC$. The former is higher than the latter for $\frac{1-\beta-\gamma+\beta\gamma}{2-\gamma} \leq SC < \frac{3+\beta-2\gamma}{4-2\gamma}$. When $\frac{3+\beta-2\gamma}{4-2\gamma} \leq SC < 1$, the contingency selling revenue is $\frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2}$, while the traditional selling revenue is $\left(1-\frac{(1-\beta)(1-\gamma)SC}{2-\gamma}\right)SC$. The former is higher than the latter for $\frac{3+\beta-2\gamma}{4-2\gamma} \leq SC < 1$.

Second, I turn to the case $SC \geq 1$. Under contingency selling, the revenue is $\frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2}$. Under traditional selling, the revenue depends on the values of parameters γ and β. When $\gamma > \tilde{\gamma}$, the traditional selling revenue is $1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$, which is lower than the contingency selling revenue.

When $\gamma \leq \tilde{\gamma}$, the traditional selling revenue is $1-\frac{(1-\beta)(1-\gamma)}{2-\gamma}$ for $SC < \widetilde{SC}$. As I show in the previous paragraph, this revenue is lower than the contingency selling revenue. When $\widetilde{SC} \leq SC < \frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}$, the traditional selling revenue is $\left(1-\frac{(1-\beta)\left((1-\gamma)SC+\gamma\right)}{2-\gamma}\right)SC$. The comparison between traditional selling and contingency selling again depends on the values of parameters γ and β.

When $\beta < \sqrt{5}-2$, the contingency selling revenue is always higher than the traditional selling revenue. When $\beta \geq \frac{1}{2}$, if SC is smaller (greater) than $\widehat{SC}$, the contingency selling revenue $\frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2}$ is higher (lower) than $\left(1-\frac{(1-\beta)\left((1-\gamma)SC+\gamma\right)}{2-\gamma}\right)SC$. It can be verified that $\widehat{SC}$ lies in the range $\left[1,\frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}\right]$. When $5-2\sqrt{6} \leq \beta < \frac{1}{2}$, if SC is smaller (greater) than $\widehat{SC}$ and $\gamma < \hat{\gamma} = \frac{3-4\sqrt{2\beta}+\beta}{1-2\sqrt{2\beta}+\beta}$, the contingency selling revenue $\frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2}$ is higher (lower) than $\left(1-\frac{(1-\beta)\left((1-\gamma)SC+\gamma\right)}{2-\gamma}\right)SC$. It can be verified that $\widehat{SC}$ lies in the range $\left[1,\frac{2-(2-\beta)\gamma}{2(1-\beta)(1-\gamma)}\right]$ for $\gamma < \hat{\gamma}$.

For $\sqrt{5}-2 \leq \beta < 5-2\sqrt{6}$, I prove that there exists a $\hat{\gamma}$ such that if SC is smaller (greater) than $\widehat{SC}$ and $\gamma < \hat{\gamma}$, the contingency selling revenue $\frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2}$ is higher (lower) than $\left(1-\frac{(1-\beta)((1-\gamma)SC+\gamma)}{2-\gamma}\right)SC$. When $\sqrt{5}-2 \leq \beta < 5-2\sqrt{6}$ and $\gamma > \frac{2-4\beta}{2-3\beta}$, the traditional selling revenue is $\left(1-\frac{(1-\beta)((1-\gamma)SC+\gamma)}{2-\gamma}\right)SC$ for $SC \in (1,2]$. It can be checked that the contingency revenue $\frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2}$ is higher than $\left(1-\frac{(1-\beta)((1-\gamma)SC+\gamma)}{2-\gamma}\right)SC$ for $SC \in (1,2]$. When $\sqrt{5}-2 \leq \beta < 5-2\sqrt{6}$ and $\gamma \leq \frac{2-4\beta}{2-3\beta}$, the highest traditional selling revenue is $\frac{(2-(2-\beta)\gamma)^2}{4(1-\beta)(1-\gamma)(2-\gamma)}$ for $SC \in (1,2]$. Thus, the difference contingency selling revenue and (highest) traditional selling revenue is $\Delta = \frac{(3+\beta-\gamma-\beta\gamma)^2}{4(2-\gamma)^2} - \frac{(2-(2-\beta)\gamma)^2}{4(1-\beta)(1-\gamma)(2-\gamma)}$. It can be verified that Δ is positive at $\gamma = \frac{2-4\beta}{2-3\beta}$ and negative at $\gamma = 0$. By checking the sign of the first order derivative, I obtain that Δ is increasing in γ for $\gamma \in \left[0, \frac{2-4\beta}{2-3\beta}\right]$. Thus, there must exist a $\hat{\gamma} \in \left[0, \frac{2-4\beta}{2-3\beta}\right]$ so that $\Delta = 0$ at $\hat{\gamma}$. For $\gamma \geq \hat{\gamma}$, contingency selling always generate higher revenue. For $\gamma < \hat{\gamma}$, contingency selling revenue is higher (lower) than traditional selling revenue if SC is smaller (greater) than $\widehat{SC}$.

In summary, contingency selling always generates higher revenue than traditional selling if $\beta < \sqrt{5}-2$ or $\gamma \geq \hat{\gamma}$. If $\beta \geq \sqrt{5}-2$ and $\gamma < \hat{\gamma}$, contingency selling (traditional selling) generates higher revenue if $SC < \widehat{SC}$ ($SC \geq \widehat{SC}$).

Appendix 12. Proof of Proposition 1.5

First, it is easy to see check that, for $SC < \frac{1}{2}$, contingency selling outperforms spot selling since $\pi_C = max\left\{2SC(1-SC), \frac{2\beta SC(2-SC)}{1+\beta}\right\} > \pi_S = SC$ for $SC < \frac{1}{2}$. Second, I turn to the case $\frac{1}{2} \leq SC < 1$, where $\pi_C = max\left\{\frac{1}{2}, \frac{2\beta SC(2-SC)}{1+\beta}\right\}$ and $\pi_S = SC$. For $\beta < \frac{1}{2}$, $\pi_C < \pi_S$. For $\beta \geq \frac{1}{2}$, π_C is greater (smaller) than π_S if $SC < \frac{3}{2} - \frac{1}{2\beta}$ ($SC \geq \frac{3}{2} - \frac{1}{2\beta}$). Third, for $SC \geq 1$, $\pi_C = max\left\{\frac{1}{2}, \frac{2\beta}{1+\beta}\right\}$ is always lower than $\pi_S = max\{1, \beta SC\}$. In summary, Proposition 5 is proved.

Appendix 13. Derivation of equilibrium with Ticket Option

When the firm sells ticket options, it sets an option price P_O and an exercise price P_E. The firm sells ticket options in the first stage when the state is uncertainty. Consumers can choose to purchase the option at P_O. In the second stage, the true state of the world (A or B) is realized. Consumers who have bought the option in the first period has the right to exercise the option by paying P_E.

Whether a consumer chooses to exercise her option depends on both the value of P_E and the realized state. When $P_E > \beta$, a consumer is only willing to exercise the option if her preferred state is realized. When $P_E \leq \beta$, a consumer is willing to exercise the option regardless of the realized state. I solve for the optimal pricing strategies in these two conditions respectively.

We first analyze the case $P_E \leq \beta$, where all the consumers anticipate that they will exercise the option in either state. For a consumer i in segment 1, her valuation for the option takes the following form.

$$\rho_i(1 - P_E) + (1 - \rho_i)(\beta - P_E)$$

Since ρ_i follows the uniform distribution on $[0, 1]$, the segment 1 consumers' valuations follow the uniform distribution on $[\beta - P_E, 1 - P_E]$. Similarly, I can show that the segment 2 consumers' valuations also follow the uniform distribution on $[\beta - P_E, 1 - P_E]$. These two consumer segments jointly generate the following aggregate demand function.

$$Q(P_O) = \begin{cases} 2, & P_O < \beta - P_E \\ \dfrac{2(1 - P_O - P_E)}{1 - \beta}, & \beta - P_E \leq P_O < 1 - P_E \\ 0, & P_O \geq 1 - P_E \end{cases}$$

Since all the consumers who purchase the option will exercise it in either state, the margin for the firm is $P_O + P_E$. The above demand function can be rewritten as follow.

$$Q(P_O + P_E) = \begin{cases} 2, & P_O + P_E < \beta \\ \dfrac{2(1 - P_O - P_E)}{1-\beta}, & \beta \le P_O + P_E < 1 \\ 0, & P_O + P_E \ge 1 \end{cases}$$

Note that this demand function is identical to the one under traditional selling in the main model. Following the same logic, I can obtain the same solution as in traditional selling.

Second, I turn to the case $P_E > \beta$. When $P_E > \beta$, a consumer is only willing to exercise the option in her preferred state. For a consumer i in segment 1, her valuation for the option takes the following form.

$$\rho_i(1 - P_E)$$

Since ρ_i follows the uniform distribution on $[0, 1]$, segment 1 consumers' valuations follow the uniform distribution on $[0, 1 - P_E]$. Similarly, I can show that segment 2 consumers' valuations also follow the uniform distribution on $[0, 1 - P_E]$. These two consumer segments jointly generate the following aggregate demand function.

$$Q(P_O) = \begin{cases} 2 - \dfrac{2P_O}{1 - P_E}, & 0 \le P_O < 1 - P_E \\ 0, & P_O \ge 1 - P_E \end{cases}$$

Since the two segments are symmetric, the demands generated in the two segments should also be the same, given any price. Thus, only half of the consumers with the option will exercise it in a particular state. In total, the firm's margin per option would be $P_O + \frac{P_E}{2}$. The revenue function takes the following form.

$$\pi(P_O, P_E) = \begin{cases} \left(2 - \dfrac{2P_O}{1 - P_E}\right)\left(P_O + \dfrac{P_E}{2}\right), & 0 \le P_O < 1 - P_E \\ 0, & P_O \ge 1 - P_E \end{cases}$$

The firm chooses P_O and P_E to maximize its revenue subject to the following constraint.

$$\begin{cases} P_E > \beta \\ 2 - \dfrac{2P_O}{1 - P_E} \leq SC \end{cases}$$

When $SC \leq 1$, the optimal option price P_O given exercise price P_E is $\frac{1}{2}(1 - P_E)(2 - SC)$. If I let P_O be $\frac{1}{2}(1 - P_E)(2 - SC)$, the revenue function can be rewritten as

$$\pi(P_E) = \frac{1}{2}SC\big(2 - SC - P_E(1 - SC)\big)$$

The revenue is decreasing in P_E for $SC \leq 1$. Thus, I obtain the following inequality.

$$\pi(P_E) < \pi(\beta) = \frac{1}{2}SC\big(2 - \beta - SC(1 - \beta)\big)$$

Note that this revenue is lower than the revenue in the case $P_E \leq \beta$, which is $\frac{1}{2}SC\big(2 - SC(1 - \beta)\big)$. Thus, the firm won't set P_E higher than β, when $SC \leq 1$.

When $SC > 1$, the optimal option price P_O given exercise price P_E is $\frac{1}{2}(1 - P_E)(2 - SC)$ if $SC < \frac{2-P_E}{2(1-P_E)}$ and $\frac{1}{4}(2 - 3P_E)$ otherwise. In the former scenario, the revenue is $\frac{1}{2}SC\big(2 - SC - P_E(1 - SC)\big)$, while the revenue is $\frac{(2-P_E)^2}{8(1-P_E)}$ in the latter scenario. In can be checked that both expression is increasing in P_E for $SC > 1$. Thus, the optimal exercise price is $P_E = 1$, which makes $SC < \frac{2-P_E}{2(1-P_E)}$ always hold. The corresponding option price is $P_O = 0$. The revenue is $\frac{1}{2}SC$. Note that the revenue in case $P_E \leq \beta$ is $\frac{1}{2}SC\big(2 - SC(1 - \beta)\big)$ for $SC < \frac{1}{1-\beta}$ and $\frac{1}{2(1-\beta)}$ otherwise. In comparison, I obtain that $\frac{1}{2}SC$ is higher than the revenue in case $P_E \leq \beta$ if $SC \geq \frac{1}{1-\beta}$ and $\beta \leq \frac{1}{2}$.

In summary, when $\beta > \frac{1}{2}$ or $SC < \frac{1}{1-\beta}$, the firm sets $P_O + P_E = 1 - \frac{SC(1-\beta)}{2}$ and earns a revenue $\frac{SC(2-SC(1-\beta))}{2}$. When $SC \geq \frac{1}{1-\beta}$ and $\beta \leq \frac{1}{2}$, the firm sets $P_O = 0$ and $P_E = 1$, and the revenue is $\frac{1}{2}SC$.

Finally, I compare the ticket option with contingency selling. It turns out that contingency selling generates higher (lower) revenue than ticket option if $SC < \overline{SC}$ ($SC \geq \overline{SC}$). The threshold $\overline{SC}$ is identical to the threshold in Proposition 2. This is because, the ticket option revenue is identical to traditional selling for $\beta > \frac{1}{2}$ or $SC < \frac{1}{1-\beta}$, and the threshold $\overline{SC}$ lies in the range $\left[0, \frac{1}{1-\beta}\right]$. When $SC \geq \frac{1}{1-\beta}$ and $\beta \leq \frac{1}{2}$, contingency selling revenue is lower than the traditional selling revenue, which is even lower than the ticket option revenue.

Overall, contingency selling outperforms ticket option for low service capacity.

Appendix 14. Derivation of the Entrepreneur's strategy for the full model

In this appendix, I solve for the entrepreneur's optimal advertising, quality, and price strategies for the full model. All the other scenarios in the paper can be viewed as modifications of the full model here.

First, I look at the traditional scenario. The entrepreneur's profit function is $\pi_T(S_T, K_T, P_T) = K_T M \left(1 - \frac{P_T}{S_T}\right) P_T - C(S_T) - ad(K_T)$ with uncertain M. The expected profit is $E[\pi_T(S_T, K_T, P_T)] = \frac{1}{2} K_T \left(1 - \frac{P_T}{S_T}\right) P_T - C(S_T) - ad(K_T)$. Facing such profit function, the entrepreneur chooses K_T, S_T, and P_T sequentially.

By differentiating $\pi_T(S_T, K_T, P_T)$ with respect to P_T and setting the derivative to zero, I obtain the optimal price is $P_T^* = \frac{S_T}{2}$. By plugging in $P_T^* = \frac{S_T}{2}$, $C(S_T) = \frac{{S_T}^2}{2}$, and $ad(K_T) = \frac{{K_T}^3}{24}$, the expected profit function can be rewritten as $E[\pi_T(S_T, K_T)] = \frac{K_T S_T}{8} - \frac{{S_T}^2}{2} - \frac{{K_T}^3}{24}$.

We differentiate the profit expectation with respect to S_T to obtain the first order condition, $\frac{K_T}{8} - S_T = 0$. Obviously, the optimal quality is $S_T^* = \frac{K_T}{8}$. By plugging in $S_T^* = \frac{K_T}{8}$, I rewrite the profit expectation as $E[\pi_T(K_T)] = \frac{{K_T}^2}{128} - \frac{{K_T}^3}{24}$. By differentiating this expression with respect to K_T, I reach the first order condition, $\frac{K_T}{64} - \frac{{K_T}^2}{8} = 0$. There are two solutions to this condition, $K_T = 0$ and $K_T = \frac{1}{8}$. The one that also satisfies the second order condition is $K_T^* = \frac{1}{8}$.

Second, I turn into the crowdfunding scenario, where the entrepreneur's profit function is given as follows.

$$\pi_C(S_C, K_C, P_C, G) = \begin{cases} K_C M\left(1 - \frac{P_C}{S_C}\right) P_C - \frac{{S_C}^2}{2} - \frac{{K_C}^3}{24}, & K_C M\left(1 - \frac{P_C}{S_C}\right) P_C \geq G \\ 0 - \frac{{K_C}^3}{24}, & K_C M\left(1 - \frac{P_C}{S_C}\right) P_C < G \end{cases}$$

Since M follows $[0, 1]$ uniform distribution, the entrepreneur's expected profit can be written as $E[\pi_C(S_C, K_C, P_C, G)] = \left(1 - \frac{GS_C}{K_C P_C (S_C - P_C)}\right) \frac{GS_C - {S_C}^3 + K_C P_C (S_C - P_C)}{2S_C} - \frac{{K_C}^3}{24}$.

By differentiating the above expression with respect to G and setting the derivative to zero, I obtain the optimal funding goal $G^* = \frac{{S_C}^2}{2}$. I then substitute $G^* = \frac{{S_C}^2}{2}$ and obtain $E[\pi_C(S_C, K_C, P_C)] = \frac{GS_C\left(2K_C P_C (S_C - P_C) + {S_C}^3\right)^2}{8K_C P_C S_C (S_C - P_C)} - \frac{{K_C}^3}{24}$.

By differentiating the above expression with respect to P_C and setting the derivative to zero, I obtain the optimal price $P_C^* = \frac{S_C}{2}$. Then, I rewrite the expected profit as $E[\pi_C(S_C, K_C)] = \frac{1}{8K_C}(K_C - 2S_C)^2 S_C - \frac{{K_C}^3}{24}$.

We then differentiate it with respect to S_C to obtain the first order condition $\frac{(K_C - 6S_C)(K_C - 2S_C)}{8K_C} = 0$. There are two S_C values that satisfy this condition. By checking the second order condition, I obtain the optimal quality level $S_C^* = \frac{K_C}{6}$. Now I can rewrite the profit expectation as $E[\pi_C(K_C)] = \frac{{K_C}^2}{108} - \frac{{K_C}^3}{24}$. I differentiate this expression with respect to K_C and get the first order condition, $\frac{K_C}{54} - \frac{{K_C}^2}{8} = 0$. There are solutions to this condition, $K_C = 0$ and $K_C = \frac{4}{27}$. The one that also satisfies the second order condition is $K_C^* = \frac{4}{27}$.

Appendix 15. Proof of Proposition 2.1

In this appendix, I solve for the entrepreneur's optimal quality and price strategy in the base model. The model here can be viewed as a special case of the full model in Chapter 2 Section 5, with the modification that the advertising is cost free. When advertising is costless, the entrepreneur will choose the maximal possible advertising level $K_T^* = K_C^* = 1$ in both the traditional and the crowdfunding entry model. Following the manner of Appendix 14, I can derive the entrepreneur's optimal strategies in the traditional scenario and crowdfunding.

In the traditional scenario, the optimal product quality, price, and the profit expectation are respectively $S_T^* = \frac{1}{8}$, $P_T^* = \frac{1}{16}$, and $E[\pi_T^*] = \frac{1}{128}$.

In crowdfunding, the optimal product quality, price, funding goal, and the profit expectation are respectively $S_C^* = \frac{1}{6}$, $P_C^* = \frac{1}{12}$, $G^* = \frac{1}{72}$, and $E[\pi_C^*] = \frac{1}{108}$.

Finally, I compare the product quality, price, and expected profit in the traditional vs. crowdfunding scenarios. It is clear that $S_C^* = \frac{1}{6} > S_T^* = \frac{1}{8}$, $P_C^* = \frac{1}{12} > P_T^* = \frac{1}{16}$, and $E[\pi_C^*] = \frac{1}{108} > E[\pi_T^*] = \frac{1}{128}$. Proposition 1 is proved.

Appendix 16. Derivation of Equation (8) and (9)

In this appendix, I solve for the optimal advertising level for a fixed quality level $\bar{S}$. The model here is similar to the full model in Appendix 14 expect that both S_T and S_C are fixed at $\bar{S}$.

First, I look at the traditional scenario. Following the same fashion of Appendix 14, I obtain the optimal price $P_T^* = \frac{\bar{S}}{2}$ and rewrite the profit expectation as $E[\pi_T(\bar{S}, K_T)] = \frac{K_T \bar{S}}{8} - \frac{\bar{S}^2}{2} - \frac{{K_T}^3}{24}$. I differentiate the profit expectation with respect to K_T to obtain the first order condition, $\frac{\bar{S}}{8} = \frac{{K_T}^2}{8}$. It follows that the optimal advertising level $K_T^* = \sqrt{\bar{S}}$. Equation (8) is derived.

Second, I turn into the crowdfunding scenario, where the entrepreneur's expected profit can be written as $E[\pi_C(\bar{S}, K_C, P_C, G)] = \left(1 - \frac{G\bar{S}}{K_C P_C(\bar{S}-P_C)}\right)\frac{G\bar{S}-\bar{S}^3+K_C P_C(\bar{S}-P_C)}{2\bar{S}} - \frac{{K_C}^3}{24}$. Following the procedure in Appendix 14, I obtain the optimal funding goal $G^* = \frac{\bar{S}^2}{2}$ and price $P_C^* = \frac{\bar{S}}{2}$. Then, I rewrite the expected profit as $E[\pi_C(\bar{S}, K_C)] = \frac{1}{8K_C}(K_C - 2\bar{S})^2\bar{S} - \frac{{K_C}^3}{24}$.

By differentiating this expression with respect to K_C, I get the following first order condition, $\frac{-{K_C}^4+{K_C}^2\bar{S}-4\bar{S}^3}{8{K_C}^2} = 0$. There exist two real-value solutions that satisfy this condition, $K_C = \sqrt{\frac{1}{2}\bar{S}\left(1-\sqrt{1-16\bar{S}}\right)}$ and $K_C = \sqrt{\frac{1}{2}\bar{S}\left(1+\sqrt{1-16\bar{S}}\right)}$. By checking the second order condition, I get $K_C^* = \sqrt{\frac{1}{2}\bar{S}\left(1+\sqrt{1-16\bar{S}}\right)}$. Equation (9) is derived.

Appendix 17. Proof of Proposition 2.2

The proof of Proposition 2.2 comes directly from the comparison of K_T^* and K_C^* shown in Appendix 14. It is straightforward to see that $K_T^* = \frac{1}{8} < K_C^* = \frac{4}{27}$. is proved.

Appendix 18. Proof of Proposition 2.3

In the traditional scenario, the expected consumer surplus takes the following form.

$$E[CS_T(K_T, S_T, P_T)] = \int_0^1 \int_{\frac{P_T}{S_T}}^1 K_T M(\theta S_T - P_T) d\theta\, dM$$

We know that $P_T^* = \frac{1}{128}$, $S_T^* = \frac{1}{64}$, $K_T^* = \frac{1}{8}$, and θ follows the uniform distribution over $[0, 1]$. Thus, I can calculate the expected consumer surplus $E[CS_T] = \frac{1}{8192}$.

In crowdfunding, the consumer surplus depends on whether the project is funded.

$$CS_C(K_C,S_C,P_C,G,M)=\begin{cases} K_C M\int_{\frac{P_C}{S_C}}^{1}(\theta S_C-P_C)\,d\theta\,, & K_C M\left(1-\frac{P_C}{S_C}\right)P_C\geq G \\ 0, & K_C M\left(1-\frac{P_C}{S_C}\right)P_C<G \end{cases}$$

By plugging in $P_C^*=\frac{1}{81}$, $S_C^*=\frac{2}{81}$, $K_C^*=\frac{4}{27}$, and $G^*=\frac{2}{6561}$, I can rewrite the formula.

$$CS_C(M)=\begin{cases} \frac{4}{27}M\int_{\frac{1}{2}}^{1}\left(\frac{2\theta}{81}-\frac{1}{81}\right)d\theta\,, & M\geq\frac{1}{3} \\ 0, & M<\frac{1}{3} \end{cases}$$

Since θ is uniformly distributed over $[0,1]$, I can further simplify the expression.

$$CS_C(M)=\begin{cases} \frac{M}{2187}, & M\geq\frac{1}{3} \\ 0, & M<\frac{1}{3} \end{cases}$$

Thus, the expected consumer surplus is $E[CS_C]=\frac{4}{19683}$. By comparison, I conclude that $E[CS_C]>E[CS_T]$.

Appendix 19. Proof of Proposition 2.4

First, it is easy to see that the entrepreneur's in the traditional scenario does not vary with respect to β. This is because the entrepreneur's expected profit only depends on the expected M which remains still.

Second, let me examine the crowdfunding scenario, where the entrepreneur's profit function is the same as in the main model. The only difference here is that M follows the uniformly distribution over $\left[\frac{1}{2}-\beta,\frac{1}{2}+\beta\right]$. As a result, the entrepreneur's expected profit can be written as

$$E[\pi_C(S_C,K_C,P_C,G)]=\left(\frac{1}{2}+\beta-\frac{GS_C}{K_CP_C(S_C-P_C)}\right)\frac{2S_C\left(G-{S_C}^2\right)+(1+2\beta)K_CP_C(S_C-P_C)}{8\beta S_C}-\frac{{K_C}^3}{24}$$

Similar to Appendix 14, I obtain $K_C^* = \frac{1}{108\beta}(1+2\beta)^3$, $S_C^* = \frac{1}{1296\beta}(1+2\beta)^4$, $P_C^* = \frac{1}{2592\beta}(1+2\beta)^4$, and $E[\pi_C] = \frac{(1+2\beta)^9}{6(216\beta)^3}$. Following the procedure in Appendix 18, I can also calculate the expected consumer surplus $E[CS_C] = \frac{(1+2\beta)^9}{6718464\beta^3}$. It is easy to check that S_C^*, P_C^*, K_C^*, $E[\pi_C]$, and $E[CS_C]$ are all increasing in β.

Appendix 20. Proof of Proposition 2.5

Here, I solve for the entrepreneur's optimal strategies in crowdfunding. Since the quality preference θ is uniformly distributed over $[0, a]$, the demand function can be written as $D(S_C, P_C) = MK_C\left(1 - \frac{P_C}{aS_C}\right)$. Correspondingly, the profit for the entrepreneur is given as follows.

$$\pi_C(K_C, S_C, P_C, G) = \begin{cases} MK_C\left(1 - \dfrac{P_C}{aS_C}\right)P - \dfrac{{S_C}^2}{2} - ad(K_C), & K_C M\left(1 - \dfrac{P_C}{aS_C}\right)P_C \geq G \\ 0 - ad(K_C), & K_C M\left(1 - \dfrac{P_C}{aS_C}\right)P_C < G \end{cases}$$

Given the distribution of M, I can derive the entrepreneur's expected profit.

$$E[\pi_C(K_C, S_C, P_C, G)] = \left(1 - \frac{aGS_C}{K_C P_C(aS_C - P_C)}\right)\frac{K_C P_C(aS_C - P_C) + aS_C\left(G - {S_C}^2\right)}{2aS_C}$$

By equating the first order derivative to zero, I obtain the optimal funding goal $G^* = \frac{2a^6}{6561}$, price $P_C^* = \frac{a^4}{81}$, product quality $S_C^* = \frac{2a^3}{81}$, and advertising level $K_C^* = \frac{4a^2}{27}$. The expected profit is $\frac{4a^6}{59049}$. Following the manner in Appendix 18, I obtain the expected consumer surplus crowdfunding $E[CS_C] = \frac{4a^7}{19683}$.

When I compare these values to those in the traditional scenario, I find that the product quality is higher in crowdfunding than in the traditional scenario if $a \geq \frac{3}{4}\left(\frac{3}{2}\right)^{1/3}$. The advertising level is higher in crowdfunding if $a \geq \frac{3}{2}\left(\frac{1}{8}\right)^{1/4}$. The advertising level is higher in crowdfunding if $a \geq \frac{3}{4}\left(\frac{3}{2}\right)^{1/2}$. The expected consumer surplus is higher in crowdfunding if $a \geq \frac{3}{4}\left(\frac{3}{2}\right)^{1/2} \approx 0.919$. Proposition 2.5 is proved.

References

1) Alexandrov A (2015) When Should Firms Expose Themselves to Risk? Manage. Sci. 61(12):3001–3008.
2) Bass FM (1969) A New Product Growth for Model Consumer Durables. Manage. Sci. 15(5):215–227.
3) Belleflamme P, Lambert T, Schwienbacher A (2013) Individual crowdfunding practices. Ventur. Cap. 15(4):313–333.
4) Binmore KG, Herrero MJ (1988) Matching and Bargaining in Dynamic Markets. Rev. Econ. Stud. 55(1):17.
5) Biyalogorsky E, Carmon Z, Fruchter GE, Gerstner E (1999) Research Note: Overselling with Opportunistic Cancellations. Mark. Sci. 18(4):605–610.
6) Biyalogorsky E, Gerstner E (2004) Contingent Pricing to Reduce Price Risks. Mark. Sci. 23(1):146–155.
7) Biyalogorsky E, Koenigsberg O (2014) The design and introduction of product lines when consumer valuations are uncertain. Prod. Oper. Manag. 23(9):1539–1548.
8) Boyd S, Vandenberghe L (2004) Convex Optimization (Cambridge University Press, Cambridge).
9) Carlton DW, Dana JD (2008) Product Variety and Demand Uncertainty: Why Markups Vary with Quality. J. Ind. Econ. 56(3):535–552.
10) Casson M (1982) The Entrepreneur: An Economic Theory (Barnes & Noble Books, Totowa, NJ).
11) Che H, Narasimhan C, Padmanabhan V (2010) Leveraging uncertainty through backorder. Quant. Mark. Econ. 8(3):365–392.
12) Chemla G, Tinn K (2018) Learning Through Crowdfunding
13) Cho SH, Tang CS (2013) Advance Selling in a Supply Chain Under Uncertain Supply and Demand. Manuf. Serv. Oper. Manag. 15(2):305–319.
14) Courty P (2003) Ticket Pricing under Demand Uncertainty*. J. Law Econ. 46(2):627–652.
15) Cui Y, Duenyas I, Şahin Ö (2014) Should Event Organizers Prevent Resale of Tickets? Manage. Sci. 60(9):2160–2179.
16) Dehez P, Jacquemin A (1975) A Note on Advertising Policy Under Uncertainty and Dynamic Conditions. J. Ind. Econ. 24(1):73.
17) Fay S, Xie J (2010) The Economics of Buyer Uncertainty: Advance Selling vs. Probabilistic Selling. Mark. Sci. 29(6):1040–1057.
18) Fourt LA, Woodlock JW (1960) Early Prediction of Market Success for New Grocery Products. J. Mark. 25(2):31.
19) Gale D (1987) Limit theorems for markets with sequential bargaining. J. Econ. Theory 43(1):20–54.
20) Gale IL, Holmes TJ (1992) The efficiency of advance-purchase discounts in the presence of aggregate demand uncertainty. Int. J. Ind. Organ. 10(3):413–437.
21) Gale IL, Holmes TJ (1993) Advance-Purchase Discounts and Monopoly Allocation of Capacity. Am. Econ. Rev. 83(1):135–146.

22) Geng X, Wu R, Whinston AB (2007) Profiting from Partial Allowance of Ticket Resale. J. Mark. 71(2):184–195.

23) Happel SK, Jennings MM (1995) The Folly of Anti-Scalping Laws. Cato J. 15(1):65–80.

24) Harris M, Raviv A (1981) A Theory of Monopoly Pricing Schemes with Demand Uncertainty. Am. Econ. Rev. 71(3):347–365.

25) Hitsch GJ (2006) An Empirical Model of Optimal Dynamic Product Launch and Exit Under Demand Uncertainty. Mark. Sci. 25(1):25–50.

26) Horowitz I (1970) A Note on Advertising and Uncertainty. J. Ind. Econ. 18(2):151.

27) Hu M, Li X, Shi M (2015) Product and Pricing Decisions in Crowdfunding. Mark. Sci. 34(3):331–345.

28) Jagpal HS, Brick IE (1982) The Marketing Mix Decision Under Uncertainty. Mark. Sci. 1(1):79–92.

29) Kahneman D, Tversky A (1979) Prospect Theory: An Analysis of Decision under Risk. Econometrica 47(2):263.

30) Karp L, Perloff JM (2005) When promoters like scalpers. J. Econ. Manag. Strateg. 14(2):477–508.

31) Krueger AB (2001) Economic Scene; Seven lessons about Super Bowl ticket prices. The New York Times (February) http://www.nytimes.com/2001/02/01/business/economic-scene-seven-lessons-about-super-bowl-ticket-prices.html?mcubz=0.

32) Lahiri A, Dey D (2013) Effects of Piracy on Quality of Information Goods. Manage. Sci. 59(1):245–264.

33) Lim WS, Tang CS (2013) Advance Selling in the Presence of Speculators and Forward-Looking Consumers. Prod. Oper. Manag. 22(3):571–587.

34) Luan YJ, Sudhir K (2010) Forecasting Marketing-Mix Responsiveness for New Products. J. Mark. Res. 47(3):444–457.

35) McMullen J, Shepherd DA (2006) Entrepreneurial action and the role of uncertainty in the theory of the entrepreneur. Acad. Manag. Rev. 31(1):132–152.

36) Möller M, Watanabe M (2010) Advance Purchase Discounts Versus Clearance Sales*. Econ. J. 120(547):1125–1148.

37) Mollick E (2014) The dynamics of crowdfunding: An exploratory study. J. Bus. Ventur. 29(1):1–16.

38) Mollick ER, Nanda R (2016) Wisdom or Madness? Comparing Crowds with Expert Evaluation in Funding the Arts. Manage. Sci. 62(6):1533–1553.

39) Moorthy KS (1984) Market Segmentation, Self-Selection, and Product Line Design. Mark. Sci. 3(4):288–307.

40) Neelamegham R, Chintagunta P (1999) A Bayesian Model to Forecast New Product Performance in Domestic and International Markets. Mark. Sci. 18(2):115–136.

41) Osborne MJ, Rubinstein A (1990) Bargaining and markets 2nd ed. (Academic Press, Inc., San Diego, California).

42) Prasad A, Stecke KE, Zhao X (2011) Advance Selling by a Newsvendor Retailer. Prod. Oper. Manag. 20(1):129–142.

43) Riedl J (2013) Crowdfunding Technology Innovation. Computer (Long. Beach. Calif). 46(3):100–103.

44) Roma P, Gal-Or E, Chen RR (2018) Reward-Based Crowdfunding Campaigns: Informational Value and Access to Venture Capital. Inf. Syst. Res. (April): isre.2018.0777.

45) Rubinstein A, Wolinsky A (1990) Decentralized Trading, Strategic Behaviour and the Walrasian Outcome. Rev. Econ. Stud. 57(1):63.

46) Sainam P, Balasubramanian S, Bayus BL (2010) Consumer Options: Theory and an Empirical Application to a Sports Market. J. Mark. Res. 47(3):401–414.

47) Shugan SM, Xie J (2000) Advance Pricing of Services and Other Implications of Separating Purchase and Consumption. J. Serv. Res. 2(3):227–239.

48) Shugan SM, Xie J (2005) Advance-selling as a competitive marketing tool. Int. J. Res. Mark. 22(3):351–373.

49) Strausz R (2017) A Theory of Crowdfunding: A Mechanism Design Approach with Demand Uncertainty and Moral Hazard. Am. Econ. Rev. 107(6):1430–1476.

50) Su X (2010) Optimal Pricing with Speculators and Strategic Consumers. Manage. Sci. 56(1):25–40.

51) Syam N, Hess JD, Yang Y (2016) Can Sales Uncertainty Increase Firm Profits? J. Mark. Res. 53(2):199–206.

52) Urban GL, Katz GM (1983) Pre-Test-Market Models: Validation and Managerial Implications. J. Mark. Res. 20(3):221–234.

53) Villas-Boas JM (1998) Product Line Design for a Distribution Channel. Mark. Sci. 17(2):156–169.

54) Viotto da Cruz J (2018) Beyond financing: crowdfunding as an informational mechanism. J. Bus. Ventur. 33(3):371–393.

55) Xie J, Shugan SM (2001) Electronic Tickets, Smart Cards, and Online Prepayments: When and How to Advance Sell. Mark. Sci. 20(3):219–243.

56) Xu T (2017) Learning from the Crowd: The Feedback Value of Crowdfunding (Vancouver, BC, V6T1Z2, Canada).

57) Yu M, Kapuscinski R, Ahn H soo (2015) Advance Selling: Effects of Interdependent Consumer Valuations and Seller’s Capacity. Manage. Sci. 61(9):2100–2117.